Bayesian Iterative Method Using Parameter Scheduling for Predicting Optimal Condition on Thermal Index Due to Air Conditioning with Minimized Power

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***Abstract*—On the basis of the Bayesian iterative method via parameter scheduling, we investigate the prediction of a set of optimal environmental variables of small-scale space by using air conditioning with minimized power. Numerical calculations clarify dynamic properties of the environmental variables, such as temperature and relative humidity at each sampling point in the present method for several realistic cases in severe summer season. We find the optimal parameter scheduling realizing the optimal environment with minimized power of the air conditioning both using air cooling and dehumidifier.**

***Index Terms*—Bayesian inference, expected a posterior estimation, indoor environment, parameter scheduling**

I. Introduction

Researchers have investigated data-driven information techniques [1], [2] due to the development of the computer technology. Researchers have developed various information methods extracting some valuable information among large-scale data observed by various sensors, such as the principal component analysis [2]. Also, researchers have developed information techniques to predict some variables on the basis of Bayesian inference [3], [4] in many research fields, such as the time-series analysis [5]-[7].

On the other hand, for three or four decades, physicists have studied information based on the analogy between Bayesian inference and statistical mechanics [8]. They have first applied statistical-mechanical methods [9], such as the mean-field theory and the replica theory to fundamental topics, such as error-correcting codes [10] and image restoration [11]. Also, one of the present authors (Y. S.) have studied the applications to science and technology, such as image restoration using plane rotator models [12], inverse halftoning [13], phase unwrapping for interferometric SAR data [14] and time-series analysis with the principal component analysis for environmental data [15]. Now, researchers have studied information technology using the sparse modeling from the statistical-mechanical viewpoint using the loopy belief propagation [16].

On the other hand, due to the occurrence of the global-scale energy and environmental problems [17], researchers and engineers have been constructing a lot of systems for power saving. Some of them [18] have been constructing systems for effective use of electric power by using the smart grids and sensor networks connecting power plants with various systems consuming power, such as each residence and industry with various scale. Then, the Ministry of Economics, Technology and Industry (METI) [19] in Japan has pointed out that power saving at each residence does not proceed until 2030 due to an increase in load of nursing for aging population, and that it delays in each small-scale industry due to the delay in introducing the system for power saving. Also, global warming has become one of the serious problems all over the world. Therefore, it has become of importance to realize many small-scale comfortable space on the basis of the appropriately assumed thermal index by making use of the air conditioner both using air cooling and dehumidifier with minimized power for the power saving systems. Previously, we constructed one of information techniques for this problem, though the optimal condition is not derived for any cases [20], [21].

Therefore, in this study, on the basis of the Bayesian inference using the expected a posterior (EAP) estimation, we construct an information technique to provide an optimal scheduling of air conditioning due to the air cooling and the dehumidifier realizing comfortable small-scale space with minimized power. For our purpose, we propose an information technique based on the Bayesian iterative method serving the environmental variables at each time as expectations averaged over the posterior probability predicted by the Bayesian iterative method

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using the EAP estimation. In this method, we estimate the posterior probability based on the Bayes-formula using the model of the true prior and the likelihood expressing the model of the transition probability. Here, we use the model prior which enhances comfortable condition based on the thermal index using the Temperature-Humidity Index (THI). Also, we use the likelihood which expresses the model of the air conditioning both using the air cooling and the dehumidifier. Further, we here introduce the model of scheduling for air conditioning. Then, in order to clarify dynamical properties of the present method, we estimate the time evolution of several variables, such as the set of the environmental variables, the thermal index using the THI and the power consumption of the air conditioning for several realistic cases. Due to numerical calculations for the cases, we find that the present method succeeds in realizing the optimal condition due to the air conditioning with the minimized power, if we appropriately use the scheduling of the model of air conditioning.

This paper is organized as follows. First, we outline the control of air conditioning via the Bayesian iterative method. Then, we show our formulation for predicting the optimal parameter scheduling to realize the optimal condition via the air conditioning with minimized power. Next, we examine dynamic properties of the Bayesian iterative method for several realistic cases. Last chapter is devoted to summary and discussion.

II. Our Project

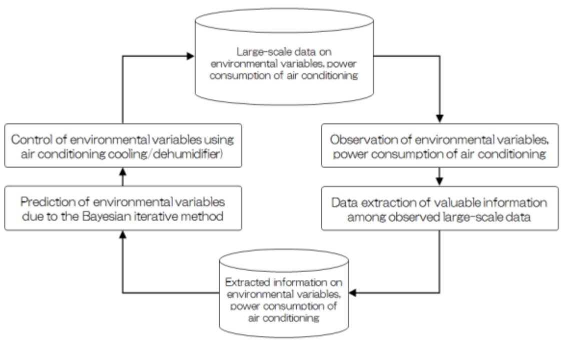


Figure 1. Framework of predicting the environmental variables based on the Bayesian inference using the EAP estimation with the use of the extracted information due to perceptron

As shown in Fig. 1, we briefly show main objective of this study on controlling air conditioning for realizing most comfortable circumstance in small areas with the minimized power due to air conditioning. Here, we study this problem for a laboratory of the National Institute of Technology, Gunma College, as a typical example of the small-scale area. Then, we use the Temperature-Humidity Index (THI) as the thermal index to estimate body sensation of human being at the area. As shown in Fig. 1, we first observe large-scale data on several indoor environmental variables, such as temperature and relative humidity at each sampling point and the electric power consumed by the air conditioning at the objective system. Then, by using the perceptron learning, we extract valuable information among large-scale data observed by some sensors, such as data loggers and power sensors. Next, based on the Bayesian iterative method with the parameter scheduling, we extract the optimal scheduling to obtain the comfortable indoor environment at the small space with the minimized power of air conditioning. Further, by making use of we control the air conditioning using the predicted variables. We repeat the above procedures until we obtain the optimal environment.

III. Bayesian Inference and Statistical Mechanics

In this chapter, we show the analogy between the Bayesian inference using the EAP estimation and statistical mechanics of the Q-Ising model.

First, the Bayesian inference using the probability theory predicts some quantities as the expectations averaged over the posterior probability estimated based on the Bayes-formula via the assumed model of the true prior and the noise probability from each original to its observed information. On the other hand, in the field of statistical mechanics, we infer the macroscopic variables averaged over the Boltzmann factor of the many-body system, such as the Q-Ising model. As seen from Fig. 2, both framework is constructed based on the probability theory. Then, Fig. 2 shows that predicting some quantities in the Bayesian inference is equivalent to estimating some macroscopic variables in statistical physics. From the correspondence in Fig. 2, we see that techniques in statistical physics, such as the mean-field theory and the replica theory established in the theory of spin glasses, can be applied to predicting some variables and to estimating statistical performance of some techniques in the Bayesian inference.

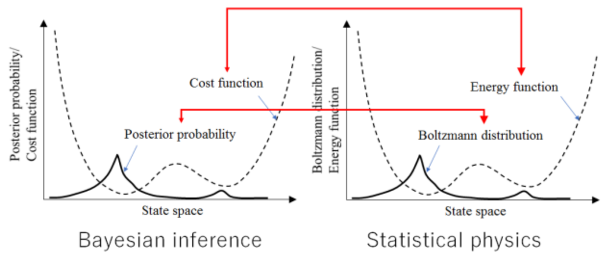


Figure 2. Correspondence between Bayesian inference and statistical physics

IV. Our Formulation

First, we consider a set of original states with most comfortable condition $\{T_s(i,t), H_s(i,t)\}$ (i=1, …, N, t=1, 2, …) which provides the optimal thermal index via the THI. Here, $T_s(i,t)$($H_s(i,t)$) is the temperature (relative humidity) of the original state at i-th point at t. Here, $T_{\min}<T_s(i,t)<T_{\max}$, $H_{\min}<H_s(i,t)<H_{\max}$. Each state $\{T_s(i,t), H_s(i,t)\}$ is generated by the assumed true prior expressed as

$$\Pr(\{T_s(i,t), H_s(i,t)\}) = \frac{1}{Z_s}\exp\left[-\beta_s \frac{\Gamma_s C_s}{f_{\mathrm{opt}}{}^2}\{f(T_s(i,t), H_s(i,t)) - f_{\mathrm{opt}}\}^2\right] \quad (1)$$

Then, we also consider the model of the true prior:

$$\Pr(\{T_s(i,t),H_s(i,t)\}) = \frac{1}{Z_s}\exp\left[-\beta_s\left\{\frac{\Gamma_s C_s}{f_{\mathrm{opt}}^{\,2}}\{f(T_s(i,t),H_s(i,t)) - f_{\mathrm{opt}}\}^2 + \frac{J_s}{N}\sum_{i<j,k}(V_s^k(i,t)-V_s^k(i,t))^2\right\}\right] \tag{2}$$

where

$$V_s^k(i,t) = \begin{cases} T_s(i,t) & (k=1) \\ H_s(i,t) & (k=2) \end{cases} \tag{3}$$

here, β_s, Γ_s, C_s and J_s are parameters set appropriately and f_{opt}= 60 as the optimal value of the THI. Then, $f(T_s(i,t), H_s(i,t))$ is the thermal index using the THI which expresses the degree of body sensation of human being. Further, the definition of the THI is expressed as

$$f(T,H) = 0.81\cdot T + 0.01\cdot H(0.99\cdot T - 14.3) + 46.3 \tag{4}$$

TABLE I. THERMAL INDEX VIA THE TEMPERATURE-HUMIDITY INDEX AND BODILY SENSATION

Temperature-humidity index	Bodily sensation
~55	cold
55~60	chilly
60~65	no feeling
65~70	comfortable
70~75	not so hot
75~80	hot
80~85	severely hot
85~	extremely hot

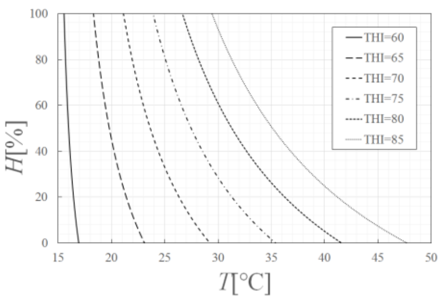


Figure 3. Curves with THI=60, 65, 70, 75, 80 and 85 described on the *T-H* plane

Then, Table I denotes the correspondence between the thermal index using the THI and the body sensation of human being. As seen from this table, we find that the human being has "no feeling" in 60<$f(T,H)$<65, and then that he/she feels "comfort" in 65<$f(T,H)$<70. The degree of discomfort becomes more serious, if $f(T,H)$ increases in 70<$f(T,H)$ and if $f(T,H)$ decreases in $f(T,H)$<60. Then, in Fig. 3, we show curves with several THI on the T-H plane. This means that the comfortable environment with $f(T,H)$=60 realized even at high temperature, if the relative humidity is low. Next, each original state $\{T_s(i,t), H_s(i,t)\}$ is converted into its observed one $\{\tau^T(i,t), \tau^H(i,t)\}$ by introducing realistic environment into the original ideal one. This means that the observed state $\{\tau^T(i,t), \tau^H(i,t)\}$ express realistic environment, if no air conditioning is used. We assume transition probability from original to observed states as

$$\Pr(\{\tau^T\},\{\tau^H\}\,|\,\{T_s\},\{H_s\}) \propto \exp\left[-\beta_s(1-C_s)H(\{\tau^T\},\{\tau^H\}\,|\,\{T_s\},\{H_s\})\right] \tag{5}$$

$$H(\{\tau^T\},\{\tau^H\}\,|\,\{T_s\},\{H_s\}) = \sum_{j=1}^{N}\left\{R_s\left(T_s(i,t)-\tau^T(i,t)\right)^2 + (1-R_s)\left(H_s(i,t)-\tau^T(i,t)\right)^2\right\} \tag{6}$$

here, we should set the parameters C_s, R_s appropriately.

Next, using the set of observed variables $\{\tau^T(i,t), \tau^H(i,t)\}$, we derive the optimal parameter scheduling of the air conditioning with air cooling and dehumidifier based on the Bayesian iterative method using the EAP estimation which provides the set of the temperature and the relative humidity and the electric power consumed by air conditioning. For our purpose, we use a set of model variables $\{T(i,t), H(i,t)\}$. Here, $T_{\min}<T(i,t)<T_{\max}$, $H_{\min}<H(i,t)<H_{\max}$, i=1,…,N, t=0,1,.... In this method, in order to derive the optimal parameter scheduling, we infer the time evolution of the environmental variables, the THI at each sampling point and the power consumed by the air conditioning as follows.

First, we consider an initial state which is expressed as $(T^*(i,0), H^*(i,0))=(\tau^T(i,0)), \tau^H(i,0))$ at t=0.

Then, we predict the set of the variables expressed as $(T^*(i,t+1),H^*(i,t+1))$ (i=1,…,N, t=0,1,2,….,N-1) using the estimated ones $(T^*(i,t), H^*(i,t))$ as

$$T^*(i,t+1) = \sum_{\{T(i,t)\}}\sum_{\{H(i,t)\}}\Pr(\{T(i,t+1),H(i,t+1)\}\,|\,\{T^*(i,t),H^*(i,t)\})\cdot T(i,t+1) \tag{7}$$

$$H^*(i,t+1) = \sum_{\{T(i,t)\}}\sum_{\{H(i,t)\}}\Pr(\{T(i,t+1),H(i,t+1)\}\,|\,\{T^*(i,t),H^*(i,t)\})\cdot H(i,t+1) \tag{8}$$

As seen from (7), (8), these two variables at t+1 are obtained as expectations which are averaged over the posterior estimated based on the Bayes-formula via the model of the true prior and the model of the transition probability from original to observed states. The explicit form of the model prior is expressed as

$$\Pr(T(i,t+1),H(i,t+1)\}) = \frac{1}{Z_m}\exp\left[-\beta\frac{\Gamma C}{f_{\mathrm{opt}}^{\,2}}\{f(T(i,t+1),H(i,t+1)) - f_{\mathrm{opt}}\}^2\right] \tag{9}$$

Also, we use another model prior whose explicit form is expressed as

$$\Pr(\{T(i,t),H(i,t)\}) = \frac{1}{Z}\exp\left[-\beta\left\{\frac{\Gamma C}{f_{\mathrm{opt}}^{\,2}}\{f(T(i,t),H(i,t)) - f_{\mathrm{opt}}\}^2 + \frac{J}{N}\sum_{i<j,k}(V^k(i,t)-V^k(i,t))^2\right\}\right] \tag{10}$$

where

$$V^k(i,t) = \begin{cases} T(i,t) & (k=1) \\ H(i,t) & (k=2) \end{cases} \tag{11}$$

If we consider the correlations between each pair of the environmental variables. Then, the explicit form of the model of the noise probability is expressed as

$$\Pr(\tau^T(i,t),\tau^H(i,t)\,|\,T(i,t+1),H(i,t+1)) \propto \exp\left[-\beta(1-C)\cdot\{R(T(i,t+1)-T^*(i,t))^2 + (1-R)(H(i,t+1)-T^*(i,t))^2\}\right] \tag{12}$$

here, β, Γ, C and J are the parameters we should appropriately. Then, we set the parameter in eq. (9) as f_{opt}=60 throughout of this research. Further, $R(t)/R$ is a parameter controlled so as to minimize the electric power of the air conditioning via the air cooling and the dehumidifier.

We repeat the procedure (2) until the optimal solution with THI=60 is approximated. If we obtain the optimal one, we stop the iteration.

At each step, we estimate the degree of the comfort human being feels based on the thermal index using the THI whose explicit form is given in (2). Then, we assue the consumed power via air conditioning as

$$\mathrm{Pow}(T(i,t),H(i,t)) = \sum_{i=1}^{n}\{\varepsilon^T\,|\,\tau^T(i,t)-T^*(i,t)\,|+\varepsilon^H\,|\,\tau^H(i,t)-H^*(i,t)\,|\} \quad (13)$$

In this study, we set to ε^T=25.66 and ε^H=20.53.

V. Performance

Here, we estimate the static property of the Bayesian inference using the EAP estimation for clarifying the roles of the model of the true prior and the transition from original to observed environmental variables and the dynamic property of the Bayesian iterative method for the realistic applications. Then, we estimate the performance for the several cases, such as (τ^T, τ^H)=(35.0[℃], 80.0[%])], (40.0[℃], 60.0[%]), (30.0[℃], 100.0[%]) at the laboratory of the National Institute of Technology, Gunma College in severe summer both with high temperature and high relative humidity.

First, we estimate the roles of the models of the true prior and the transition probability in the Bayesian inference using the EAP estimation via the numerical calculations for the realistic cases, if we set to β=1, Γ=1, $R(t)$=0.3 and J=0. Also, due to the rigorous proof via an inequality, we clarify that the lower bound of the mean square error between the original and predicted environmental variables is realized, if we have information on the assumed true prior and the noise probability from the original to observed variables. Then, the numerical calculations find that the Bayesian inference via the EAP estimation predicts most comfortable condition based on the thermal index using the THI at C=1, and that the method predicts the observed variables (τ^T, τ^H) at C=0. Also, we find that the Bayesian inference using the EAP estimation. Then, it is located between the position of the observed state (τ^T, τ^H) and the curve with $f(T,H)$=f_{opt}, if the parameter C is set between null and unity. (Fig. 4)

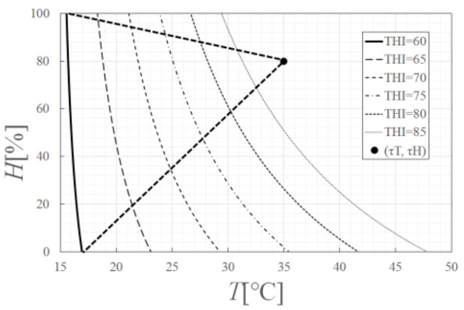


Figure 4. Locations of the predicted states obtained by the Bayesian inference using the EAP estimation for (τ^T, τ^H).

Here, we estimate the dynamic property of the Bayesian iterative method from time evolution of the environmental variables, if $R(t)$ is set to be steady. In this case, we find that the thermal index via the THI and the consumed power of the air conditioning depend on the parameter $R(t)$. Then, the results suggest that tuning the scheduling of $R(t)$ is important for realizing the optimum of the thermal index via the THI at each sampling point and for minimizing the consumed power of the air conditioning. So, we search the optimal scheduling $R(t)$ of air conditioning for realizing the optimal value of the THI at each sampling point. As shown in Figs. 5, 6(a), (b), 7, the numerical calculations clarify that the optimal value of the thermal index via the THI is realized most smoothly by means of the air conditioning with the minimized power, if we tune the parameter $\{R(t)\}$ as $\{0,1,1,1,1,\ldots\}$ among the candidates: $\{R(t)\}=\{1,1,1,1,\ldots\},\{0,1,1,1,1,\ldots\},\{0,0,1,1,1,\ldots\},\{0,0,0,1,1,\ldots\},\{0,0,0,0,1,\ldots\}$. These results indicate that the Bayesian iterative method succeeds in extracting optimal parameter scheduling realizing most comfortable environment at each sampling point by means of the air conditioning with the minimized power.

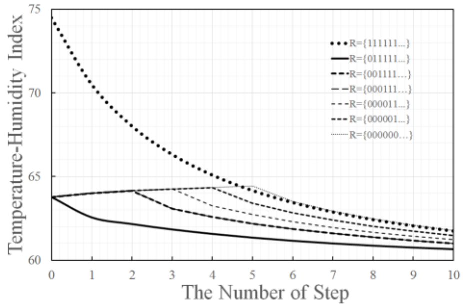
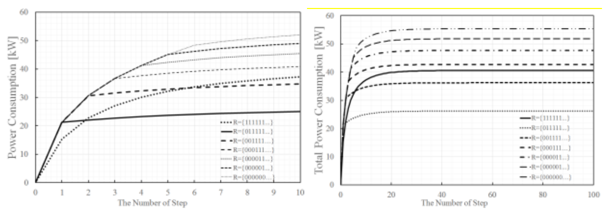


Figure 5. Time evolution of the thermal index using the temperature-humidity index at a sampling point due to the Bayesian iterative method using parameter scheduling.



(a) (b)

Figure 6. Time evolution of the power consumption due to the model of air conditioning via air cooling and dehumidifier

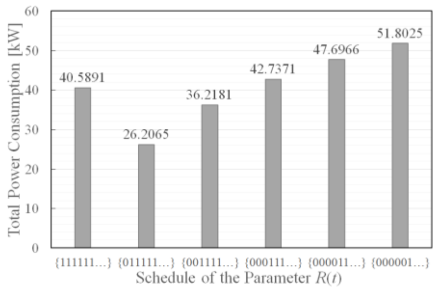


Figure 7. Total power consumption due to the air conditioning via the air conditioning and the dehumidifier depending on the parameter scheduling

VI. Summary and Discussion

In above chapters, based on the analogy between the Bayesian inference and statistical mechanics, we have constructed the information techniques for controlling the air conditioning with air cooling and dehumidifier by using the Bayesian iterative method with the parameter scheduling. In this study, we have constructed the present method based on statistical mechanics of the Q-state Ising model to predict the model of the parameter scheduling leading the optimal value of the thermal index via the THI with the power consumption due to the air conditioning via the air cooling and the dehumidifier. Here, we have considered the model of the true prior enhancing the optimal value of the thermal index via the THI at the sampling points and the model of the transition probability from the original to the observed environments expressed as the temperature and the relative humidity. Then, we have here examined the time evolution of the thermal index using the THI and the power consumption due to the air conditioning. We have obtained the results that the present method succeeds in predicting the optimal parameter scheduling which realizes the optimal value of the thermal index via the THI most smoothly with the minimized value of the electric power consumed by the air conditioning. For instance, we have found that the present method succeeds in predicting the parameter scheduling of air conditioning which realizes the optimal variables of the THI most smoothly with the minimized power with the minimized power of the air conditioning, if we carry out air cooling at first step and then dehumidifier at the following steps.

As future problems, it is important to apply the present system to realistic applications for various small-scale systems.

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