State Feedback PI-Controller for Robust Stability Analysis and Stabilization of Input-Delayed Systems

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***Abstract*—An approach is proposed in this paper to derive a delay-dependent robust stabilization criterion for uncertain input-delayed system in Linear Matrix Inequality (LMI) framework using Proportional-Integral (PI) controller. By adding integral control action for the state feedback stabilization the degree of freedom of the controller increases as a result of which the robustness increases. The matrix variables are involved in the Lyapunov-Krasovskii approach for deriving a less conservative stabilization criterion. The obtained criterion is validated by a well-known example with some existing results.**

***Index Terms*—time-delay system, PI controller, Lyapunov-Krasovskii functional**

I. Introduction

Time delay is an unavoidable phenomenon. This may occur during the transmission of information from a remotely located plant to the controller and the controller to the plant [1], [2]. It encounters in various practical systems, power systems, network control systems, economical system, biological systems etc. Its presence in the system is the major cause of performance degradation and system instability [3]–[5]. Due to the presence of this delay in the closed loop system, the analysis and controller design become challenging for control designers. It becomes more challenging when the delay is considered to be time-varying in nature [6], [7]. Many stability analysis and controller design approaches for time-delay systems have been reported in the literature, [8]–[16] and references therein. According to the dependence of the delay, the stability or stabilization criteria are generally categorized into two types, (i) delay-dependent criteria, (ii) delay-independent criteria [5], [17]. The former criteria take the size of the delay into account and the later can be applied to delays with arbitrary size. The delay-dependent analysis tends to be less conservative than the delay-independent one because it uses the information on the length of delay, especially when the delay is small [7].

For stability analysis and stabilization problem of time delay systems, time domain approaches are widely used because of their ease handling of uncertainties, nonlinearities than that of frequency domain approaches such as matrix pencil method and frequency sweeping method [18]. In time-domain approaches, two widely used techniques such as Lyapunov-Razumikhin (LR) functional approach and Lyapunov-Krasovskii (LK) functional approach [19]. It is well known that the LK based approach is less conservative than that of LR approach as it involves the delay information in the functional [4], [13]. Approaches proposed in [4], [20] by applying some bounding for some cross terms in the transformed system become conservative because the additional terms in the dynamics of the system affects the stability [4], [20]. A descriptor model transformation approach with some bounding techniques for some cross terms is proposed in [12], [20]. The proposed approach gives approach less conservative results by reducing the gap of classical model transformation approaches. For further reduction of conservatism, Jensen inequality based approach is proposed in [5], [19]. In the proposed approach, the authors avoid to use the model transformation and bounding techniques. Matrix variables are involved in [13], [14] for deriving the stability and stabilization criteria to reduce conservatism.

Recently, controller design for uncertain time-delay system has got considerable attention. A lot of research articles are published in this domain; the Riccati equation approach [21], the LMI approach [3], [22]–[24], the H_∞ control theory [25], Sliding mode control theory [26], pole-placement technique [11], [27], model reduction method (i.e. transformation delay system to an equivalent delay free system) [28]. A robust static state feedback controller is designed in [29], [30] for uncertain time delay system. The proposed controller structure [29], [30]

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is simple and easy for implementation. In [10], [31], a static state feedback controller of structure $u(t) = Kz(t)$, where $z(t) = x(t) + \int_{t-\tau_0}^{t} e^{A(t-s-\tau_0)} B_1 u(s) ds$ is used. The proposed controller in [10], [31] is difficult to design and implement because of its integral part. In [30], a simple static state feedback controller is used with Jensen's inequality based tighter delay bound to obtain less conservative result.

This paper includes a design method for state feedback controller using an integral control action with proportional control action (alone it is memory less controller or static controller). This type of composite structure of the control action increases the degree of freedom as a result of which the robustness improves. To obtain less conservative result some important things are taken care such as: (i) Jensen's inequality is used with tighter delay bound for integral approximation in the derivative of the functional, (ii) a simple linearization technique is used, (iii) more state variable are included with the system dynamics to obtain the delay-dependent LMI criteria. To show the ability of the deigned PI-controller two academic examples are considered [10], [27], [31].

The system description and some preliminary ideas are given in Section II. In Section III describes the stability analysis and controller designed for uncertain time-delay system. Section IV presents two numerical examples to show the effectiveness of the stabilization criterion and the conclusion of the paper is presented in Section V.

II. System Description and Preliminaries

Consider a linear uncertain system with input delay

$$\begin{gathered} \dot{x}(t) = A_0(t)x(t) + B_1(t)u(t) + B_2(t)u(t-\tau(t)) \\ x(t) = \phi(t),\ t \in [-\tau_2, -\tau_1] \end{gathered} \tag{1}$$

where $\tau(t)$ is the input delay assume to be time-varying in nature and satisfying $0 \le \tau_1 \le \tau(t) \le \tau_2$, $\bar{\tau} = \tau_2 - \tau_1$ and $0 \le \dot{\tau}(t) \le \mu$; $x(t) \in R^n$ is the state vector of the system; $u(t) \in R^n$ is the control input; $u(t-\tau(t))$ is the delayed control input to the system and ϕ is a continuously differentiable initial function. The objective in this paper is to design a state-feedback stabilizing PI controller for system (1). The controller structure is as follows:

$$u(t) = K_p x(t) + K_I \int_0^t x(\theta) d\theta \tag{2}$$

where K_p and K_I are the control gains to be designed such that the controller will be able to stabilize the system. To represent the controller in form of simple static state feedback controller structure, let us consider

$$\dot{z}(t) = x(t) \tag{3}$$

and

$$\bar{x}(t) = \begin{bmatrix} x^T(t) & z^T(t) \end{bmatrix}^T \tag{4}$$

The control input (2) can be written as:

$$u(t) = \begin{bmatrix} K_p & K_I \end{bmatrix} \bar{x}(t) \tag{5}$$

The augmented form of (1) can be

$$\dot{\bar{x}}(t) = \bar{A}_0 \bar{x}(t) + \bar{B}_1 K \bar{x}(t) + \bar{B}_2 K \bar{x}(t-\tau(t)) \tag{6}$$

where

$$\bar{A}_0 = \begin{bmatrix} (A_0 + \Delta A_0(t)) & 0 \\ I & 0 \end{bmatrix}, \bar{B}_0 = \begin{bmatrix} (B_1 + \Delta B_1(t)) \\ 0 \end{bmatrix}$$

$$\bar{B}_1 = \begin{bmatrix} (B_2 + \Delta B_2(t)) \\ 0 \end{bmatrix}, K = \begin{bmatrix} K_p & K_I \end{bmatrix}$$

The following lemmas will be used to derive main stability criterion.

Lemma 1 (Schur complement :2011) For given constant matrices X_1, X_2 and X_3 of appropriate dimensions, where $X_1^T = X_1$ and $X_2^T = X_2$, then

$$X_1 + X_3^T X_2^{-1} X_3 < 0,$$

if and only if $\begin{bmatrix} X_1 & X_3^T \\ X_3 & -X_2 \end{bmatrix} < 0$ or $\begin{bmatrix} -X_2 & X_3 \\ X_3^T & X_1 \end{bmatrix} < 0$.

Lemma 2: For any constant matrix $R_2 > 0$, arbitrary matrices M_1, M_2, N_1, N_2, the following inequalities satisfy:

$$\begin{aligned} & -\bar{\tau}^{-1} \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(\theta) R_2 \dot{x}(\theta) d\theta \\ & \le \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \end{bmatrix}^T \left\{ \bar{\tau}^{-1} \begin{bmatrix} M_1 + M_1^T & -M_1 + N_1^T \\ * & -N_1 - N_1^T \end{bmatrix} \right. \\ & \quad \left. + \rho \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} R_2^{-1} \begin{bmatrix} M_1 \\ N_1 \end{bmatrix}^T \right\} \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \end{bmatrix} \\ & + \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau(t)) \end{bmatrix}^T \left\{ \bar{\tau}^{-1} \begin{bmatrix} M_2 + M_2^T & -M_2 + N_2^T \\ * & -N_2 - N_2^T \end{bmatrix} \right. \\ & \quad \left. + (1-\rho) \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} R_2^{-1} \begin{bmatrix} M_2 \\ N_2 \end{bmatrix}^T \right\} \begin{bmatrix} x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix} \end{aligned} \tag{7}$$

where $\rho = \dfrac{\tau(t) - \tau_1}{\bar{\tau}}, 0 \le \rho \le 1$

Proof: Using Lemma 1 of [32], the following bounds can be written:

$$\begin{aligned} & -\bar{\tau}^{-1} \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(\theta) R_2 \dot{x}(\theta) d\theta \\ & \le \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \end{bmatrix}^T \left\{ \bar{\tau}^{-1} \begin{bmatrix} M_1 + M_1^T & -M_1 + N_1^T \\ * & -N_1 - N_1^T \end{bmatrix} \right. \\ & \quad \left. + \rho \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} R_2^{-1} \begin{bmatrix} M_1 \\ N_1 \end{bmatrix}^T \right\} \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \end{bmatrix} \end{aligned} \tag{8}$$

$$-\bar{\tau}^{-1}\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(\theta)R_2\dot{x}(\theta)d\theta$$

$$\leq \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau(t)) \end{bmatrix}^T \left\{ \bar{\tau}^{-1}\begin{bmatrix} M_2+M_2^T & -M_2+N_2^T \\ * & -N_2-N_2^T \end{bmatrix} + (1-\rho)\begin{bmatrix} M_2 \\ N_2 \end{bmatrix} R_2^{-1} \begin{bmatrix} M_2 \\ N_2 \end{bmatrix}^T \right\} \begin{bmatrix} x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix} \quad (9)$$

Combining both (8) and (9), one obtains (7).

Lemma 3: For any arbitrary matrices S_1, S_2, S_3, S_4 and S_5 the following condition holds:

$$2\xi^T(t)\begin{bmatrix} S_1^T & S_2^T & S_3^T & S_4^T & S_5^T \end{bmatrix}^T \left\{-\dot{\bar{x}}(t)+\bar{A}_0\bar{x}(t)+\bar{B}_1K\bar{x}(t)+\bar{B}_2K\bar{x}(t-\tau(t))\right\}=0 \quad (10)$$

where

$$\xi(t)=\begin{bmatrix} \bar{x}^T(t) & \bar{x}^T(t-\tau_1) & \bar{x}^T(t-\tau(t)) & \bar{x}^T(t-\tau_2) & \dot{\bar{x}}^T(t) \end{bmatrix}^T$$

Proof: Using (6), one can write

$$\left\{-\dot{\bar{x}}(t)+\bar{A}_0\bar{x}(t)+\bar{B}_1K\bar{x}(t)+\bar{B}_2K\bar{x}(t-\tau(t))\right\}=0 \quad (11)$$

One obtains (10) by multiplying any factor with zero term (11).

III. State Feedback Stability Analysis

In sequel to the previous section, a robust stability criterion for systems with norm bounded uncertainty with input delay is presented as follows.

Theorem 1: System (6) is stable if there exist matrices $P>0$, $Q_j>0$, $j=1,\ldots,4$, $R_i>0$, and arbitrary matrices S_k, $k=1,\ldots,5$, $\bar{M}_i$, $\bar{N}_i$, $i=1,2$, that satisfy the following LMI:

$$\begin{bmatrix} \Theta & \phi_l \\ * & -R_2 \end{bmatrix}<0, \quad l=1,2 \quad (12)$$

where

$$\phi_1=\begin{bmatrix} 0 & M_1^T & N_1^T & 0 & 0 \end{bmatrix}^T,$$

$$\phi_2=\begin{bmatrix} 0 & M_2^T & N_2^T & 0 & 0 \end{bmatrix}^T, \Theta=[\Theta_{ij}]_{i,j=1,\ldots,5},$$

$$\Theta_{11}=\sum_{i=1}^{3}Q_i-R_1+S_1\bar{A}_0+\bar{A}_0^TS_1^T+S_1\bar{B}_1K+K^T\bar{B}_1^TS_1^T,$$

$$\Theta_{12}=R_1+\bar{A}_0^TS_2^T+K^T\bar{B}_1^TS_2^T,$$

$$\Theta_{13}=S_1\bar{B}_2K+\bar{A}_0^TS_3^T+K^T\bar{B}_1^TS_3^T,$$

$$\Theta_{14}=\bar{A}_0^TS_4^T+K^T\bar{B}_1^TS_4^T,$$

$$\Theta_{15}=P-S_1+\bar{A}_0^TS_5^T+K^T\bar{B}_1^TS_5^T,$$

$$\Theta_{22}=-(Q_1-Q_4)-R_1+\bar{\tau}^{-1}\left[M_1+M_1^T\right],$$

$$\Theta_{23}=S_2\bar{B}_2K+\bar{\tau}^{-1}\left[-M_1+N_1^T\right],\Theta_{24}=0,\Theta_{25}=-S_2,$$

$$\Theta_{33}=-\sum_{i=3}^{4}(1-\mu)Q_i+S_3\bar{B}_2K+K^T\bar{B}_2^TS_3^T+\bar{\tau}^{-1}\left[-N_1-N_1^T\right]+\bar{\tau}^{-1}\left[M_2+M_2^T\right],$$

$$\Theta_{34}=K^T\bar{B}_2^TS_4^T+\bar{\tau}^{-1}\left[-M_2+N_2^T\right],$$

$$\Theta_{35}=-S_3+K^T\bar{B}_2^TS_5^T,$$

$$\Theta_{44}=-Q_2+\bar{\tau}^{-1}\left[-N_2-N_2^T\right],\Theta_{45}=-S_4,$$

$$\Theta_{55}=\tau_1^2R_1+R_2-S_5-S_5^T.$$

Proof: Let us consider a LK functional as follows:

$$V(t)=\bar{x}^T(t)P\bar{x}(t)+\sum_{i=1}^{2}\int_{t-\tau_i}^{t}\bar{x}^T(\theta)Q_i\bar{x}(\theta)d\theta + \int_{t-\tau(t)}^{t}\bar{x}^T(\theta)Q_3\bar{x}(\theta)d\theta+\int_{t-\tau(t)}^{t-\tau_1}\bar{x}^T(\theta)Q_4\bar{x}(\theta)d\theta + \tau_1\int_{t-\tau_1}^{t}\int_{\theta}^{t}\dot{\bar{x}}^T(\varphi)R_1\dot{\bar{x}}(\varphi)d\varphi d\theta + \bar{\tau}^{-1}\int_{t-\tau_2}^{t-\tau_1}\int_{\theta}^{t}\dot{\bar{x}}^T(\varphi)R_2\dot{\bar{x}}(\varphi)d\varphi d\theta. \quad (13)$$

Differentiating the energy functional (13) with respect to time along the state trajectory of (6) is

$$\dot{V}(t)=2\bar{x}^T(t)P\dot{\bar{x}}(t)+\sum_{i=1}^{3}\bar{x}^T(t)Q_i\bar{x}(t) -\sum_{i=3}^{4}(1-\mu)\bar{x}^T(t-\tau(t))Q_i\bar{x}(t-\tau(t)) -\bar{x}^T(t-\tau_1)(Q_1-Q_4)\bar{x}(t-\tau_1)-\bar{x}^T(t-\tau_2)Q_2\bar{x}(t-\tau_2) +\dot{\bar{x}}^T(t)(\tau_1^2R_1+R_2)\dot{\bar{x}}(t)-\tau_1\int_{t-\tau_1}^{t}\dot{\bar{x}}^T(\theta)R_1\dot{\bar{x}}(\theta)d\theta -\bar{\tau}^{-1}\int_{t-\tau_2}^{t-\tau_1}\dot{\bar{x}}^T(\theta)R_2\dot{\bar{x}}(\theta)d\theta. \quad (14)$$

The stability of the (6) can be analyzed by checking ˙V (t) is less than zero or not, the R.H.S. of (14) is added to (10). Then, it becomes

$$2\xi^T(t)\begin{bmatrix} S_1^T & S_2^T & S_3^T & S_4^T & S_5^T \end{bmatrix}^T \left\{-\dot{\bar{x}}(t)+\bar{A}_0\bar{x}(t)+\bar{B}_1K\bar{x}(t)+\bar{B}_2K\bar{x}(t-\tau(t))\right\} +2\bar{x}^T(t)P\dot{\bar{x}}(t)+\sum_{i=1}^{3}\bar{x}^T(t)Q_i\bar{x}(t) -\sum_{i=3}^{4}(1-\dot{\tau}(t))\bar{x}^T(t-\tau(t))Q_i\bar{x}(t-\tau(t)) -\bar{x}^T(t-\tau_1)(Q_1-Q_4)\bar{x}(t-\tau_1)-\bar{x}^T(t-\tau_2)Q_2\bar{x}(t-\tau_2) +\dot{\bar{x}}^T(t)(\tau_1^2R_1+R_2)\dot{\bar{x}}(t)-\tau_1\int_{t-\tau_1}^{t}\dot{\bar{x}}^T(\theta)R_1\dot{\bar{x}}(\theta)d\theta -\bar{\tau}^{-1}\int_{t-\tau_2}^{t-\tau_1}\dot{\bar{x}}^T(\theta)R_2\dot{\bar{x}}(\theta)d\theta. \quad (15)$$

Approximating the two integral terms in the RHS of (15) using Lemma 2, (15) can be written as

$$\xi^T(t)\left\{\Theta+\rho\phi_1R_2^{-1}\phi_1^T+(1-\rho)\phi_2R_2^{-1}\phi_2^T\right\}\xi(t) \quad (16)$$

Equation (14) is polytope of matrices and is negative definite if it's two certain vertices are negative definite individually. Then, the stability requirement can be written as:

$$\Theta+\phi_lR_2^{-1}\phi_l^T<0, \quad l=1,2. \; \Theta+\phi_lR_2^{-1}\phi_l^T<0, \quad l=1,2 \quad (17)$$

Finally, using Schur Complement on (17), one obtains (10).

IV. State Feedback Stabilization

To obtain the controller parameters of the PI controller, the above stability criterion is extended for stabilization criterion. The following robust stabilization criterion for uncertain input delayed system is presented.

Theorem 2: System (6) is stable if, for arbitrarily chosen λ, β, γ and α, there exist matrices $\bar{P}>0$, $\bar{Q}_j>0$, $j=1,\ldots,4$, $\bar{R}_i>0$, and arbitrary matrices $\bar{S}_1$, $\bar{M}_i$, $\bar{N}_i$, $i=1,2$, that satisfy the following LMI:

$$\begin{bmatrix} \bar{\Theta} & \bar{\phi}_l \\ * & -\bar{R}_2 \end{bmatrix} < 0, \quad l=1,2 \tag{18}$$

where

$$\bar{\phi}_1 = \begin{bmatrix} 0 & \bar{M}_1^T & \bar{N}_1^T & 0 & 0 \end{bmatrix}^T,$$

$$\bar{\phi}_2 = \begin{bmatrix} 0 & \bar{M}_2^T & \bar{N}_2^T & 0 & 0 \end{bmatrix}^T,$$

$$\bar{\Theta} = [\bar{\Theta}_{ij}]_{i,j=1,..,5} \quad with$$

$$\Theta_{11} = \sum_{i=1}^{3} \bar{Q}_i - \bar{R}_1 + \bar{A}_0\bar{S}_1^T + \bar{S}_1\bar{A}_0^T + \bar{B}_1\bar{Y} + \bar{Y}^T\bar{B}_1^T,$$

$$\Theta_{12} = \bar{R}_1 + \lambda\bar{S}_1\bar{A}_0^T + \lambda\bar{Y}^T\bar{B}_1^T,$$

$$\Theta_{13} = \bar{B}_2\bar{Y} + \beta\bar{S}_1\bar{A}_0^T + \beta\bar{Y}^T\bar{B}_1^T,$$

$$\Theta_{14} = \gamma\bar{S}_1\bar{A}_0^T + \gamma\bar{Y}^T\bar{B}_1^T,$$

$$\Theta_{15} = \bar{P} - \bar{S}_1^T + \alpha\bar{S}_1\bar{A}_0^T + \alpha\bar{Y}^T\bar{B}_1^T,$$

$$\Theta_{22} = -\left(\bar{Q}_1 - \bar{Q}_4\right) - \bar{R}_1 + \bar{\tau}^{-1}\left[\bar{M}_1 + \bar{M}_1^T\right],$$

$$\Theta_{23} = \lambda\bar{B}_2\bar{Y} + \bar{\tau}^{-1}\left[-\bar{M}_1 + \bar{N}_1^T\right], \Theta_{24} = 0, \Theta_{25} = -\lambda\bar{S}_1^T,$$

$$\Theta_{33} = -\sum_{i=3}^{4}(1-\mu)\bar{Q}_i + \beta\bar{B}_2\bar{Y} + \beta\bar{Y}^T\bar{B}_2^T + \bar{\tau}^{-1}\left[-\bar{N}_1 - \bar{N}_1^T\right] + \bar{\tau}^{-1}\left[\bar{M}_2 + \bar{M}_2^T\right],$$

$$\Theta_{34} = \gamma\bar{Y}^T\bar{B}_2^T + \bar{\tau}^{-1}\left[-\bar{M}_2 + \bar{N}_2^T\right],$$

$$\Theta_{35} = -\beta\bar{S}_1^T + \alpha\bar{Y}^T\bar{B}_2^T,$$

$$\Theta_{44} = -\bar{Q}_2 + \bar{\tau}^{-1}\left[-\bar{N}_2 - \bar{N}_2^T\right], \Theta_{45} = -\gamma\bar{S}_1^T,$$

$$\Theta_{55} = \tau_1^2\bar{R}_1 + \bar{R}_2 - \alpha\bar{S}_1^T - \alpha\bar{S}_1,$$

$$\bar{S}_1 = S_1^{-1}, \bar{P} = \bar{S}_1 P\bar{S}_1^T, \bar{Q}_i = \bar{S}_1 Q_i\bar{S}_1^T, i=1,...,4,$$

$$\bar{M}_j = \bar{S}_1 M_j\bar{S}_1^T, \bar{S}_1 N_j\bar{S}_1^T = \bar{N}_j, j=1,2, \bar{Y} = K\bar{S}_1^T.$$

Proof: For controller design or stabilization criterion, the proof of derived robust stability criterion, i.e Theorem 1 can be referred. And the nonlinear terms in (12) can be eliminated by considering S_2, S_3, S_4 and S_5 as:

$$S_2 = \lambda S_1, S_3 = \beta S_1, S_4 = \gamma S_1, S_5 = \alpha S_1.$$

and then pre- and post-multiplying (12) by

$$diag\left\{S_1^{-1} \quad S_1^{-1} \quad S_1^{-1} \quad S_1^{-1} \quad S_1^{-1} \quad S_1^{-1}\right\}$$

and its transpose respectively, and subsequently adopting the change of variables

$$\bar{S}_1 = S_1^{-1}, \bar{P} = \bar{S}_1 P\bar{S}_1^T, \bar{M}_i = \bar{S}_1 M_i\bar{S}_1^T, \bar{N}_i = \bar{S}_1 N_i\bar{S}_1^T,$$

$$i=1,2, \bar{Q}_j = \bar{S}_1 Q_j\bar{S}_1^T, j=1,\ldots,4, \bar{Y} = K\bar{S}_1^T.$$

One obtains (18).

To verify the above criterion proposed in this section, two numerical examples are considered in the next section.

V. Numerical Examples

Example 1: Consider a system of (1) with [10]

$$\dot{x}(t) = (A_0 + \Delta A_0)x(t) + B_2 u(t-\tau(t)), t \geq 0,$$

$$x(0) = x_{0,} u(t) = \phi(t), t \in [-0.2, 0],$$

where

$$A_0 = \begin{bmatrix} 0 & 1 \\ -1.25 & -3 \end{bmatrix}, \Delta A_0 = \begin{bmatrix} 0 & 0 \\ q & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, |q| \leq \eta.$$

Using Theorem 2, the maximum value of η (η_{max}) is achieved to be 28.7690. The tuning parameters (λ, β, γ and α) are tuned at 2.5107, -4.4885, 2.9221 and 0.9954 respectively by a controller $K = \begin{bmatrix} K_p & K_I \end{bmatrix}$, where $K_p = \begin{bmatrix} -33.1133 & -4.7441 \end{bmatrix}$ and $K_I = \begin{bmatrix} -0.0397 & -0.0008 \end{bmatrix}$, The designed controller is more robust than the existing controllers proposed in [10], [31]. To search the tuning parameters (λ, β, γ and α), *fminsearch* function of MATLAB is used. A comparative analysis is presented in Table I. The simulation result (norm of the states of the system) is presented with $x(t) = [5, -2, -3, -1]$, $t \in [-28.7690, 0]$ as initial condition in Fig. 1 using the PI-type controller for $\eta_{max} = 28.7690$. The presented result in Fig. 1 shows that the states are stable.

TABLE I. Comparison of Robustness η_{max}

Approach	η_{max}	Structure of $u(t)$
[31]	7.2568	$u(t) = Kz(t)$ where $z(t) = x(t) + \int_{t-\tau_0}^{t} e^{A(t-s-\tau_0)} B_1 u(s) ds$
[10]	10.8485	$u(t) = Kz(t)$ where $z(t) = x(t) + \int_{t-\tau_0}^{t} e^{A(t-s-\tau_0)} B_1 u(s) ds$
[30]	19.6688	$u(t) = Kx(t)$
Theorem 2	28.7690	$u(t) = K_p x(t) + K_I \int_0^t x(\theta) d\theta$ where $K_p = \begin{bmatrix} -33.1133 & -4.7441 \end{bmatrix}$ and $K_I = \begin{bmatrix} -0.0397 & -0.0008 \end{bmatrix}$

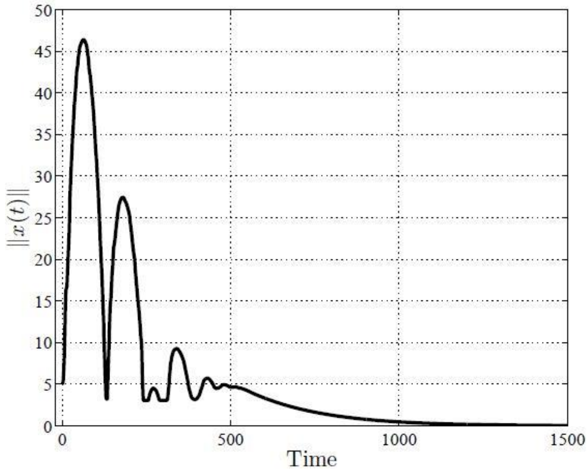

Figure 1. Variation of norm of the state vector with respect to time for Example 1

Example 2: Consider another system of (1) with [10]

$$\dot{x}(t) = (A_0 + \Delta A_0)x(t) + B_1 u(t) + B_2 u(t-\tau(t)), t \geq 0,$$

$$x(0) = x_0, u(t) = \phi(t), t \in [-0.4, 0],$$

where

$$A_0 = \begin{bmatrix} 0 & 0 \\ 1 & -5 \end{bmatrix}, \Delta A_0 = \begin{bmatrix} q & 0 \\ 0 & q \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |q| \leq \eta.$$

In this case, the η_{max} is obtained using Theorem 2 to be 1.8524. The tuning parameters λ, β, γ and α are tuned at 1.2887, 0.1741, 0.3952 and 2.9147 respectively by a controller $K = \begin{bmatrix} K_p & K_I \end{bmatrix}$, where $K_p = \begin{bmatrix} -2.3661 & -0.0035 \end{bmatrix}$ and $K_I = \begin{bmatrix} -0.0247 & -0.0074 \end{bmatrix}$, which is also more robust than the existing controllers in [10], [31]. A comparison with the existing results are presented in Table II.

TABLE II. COMPARISON OF ROBUSTNESS η_{max}

Approach	η_{max}
[10]	0.5998
[31]	1.4120
Theorem 2	1.8524

VI. CONCLUSION

In this paper, an improved robust delay-dependent stabilizing criterion has been obtained by using a PI controller. By adding the integral control action with simple memoryless controller, the degree of the freedom of the controller is increased. The dimension of the search space is also increased. As a result of which the robustness is increased. To eliminate the nonlinear terms in stabilization criterion, a simple linearization approach is adopted to obtain LMI. Finally, two numerical examples are considered to show the effectiveness of the criterion. In both the cases, the present approach is more robust than the existing results.

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