Active Distribution Network Optimization Based on Mixed Integer Linear Programming

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***Abstract*—Various distribution network optimization techniques considering Distributed Generations (DGs) are reviewed in this paper, and a new optimization method based on Mixed Integer Linear Programming (MILP) is proposed. The proposed methodology optimizes the capacity of DGs and the optimal feeder line capacity simultaneously by a cost/benefit analysis. Besides, the line loss in distribution network is explicitly analyzed by using four different methods in the paper. For simplicity, the line loss can be appropriately simplified as a quadratic function of difference of voltage phase angle. Then it is further linearized by using different linearization strategies and then compared with the results by using Mixed Integer Nonlinear Programming (MINLP) method. Finally, the proposed active distribution network planning model with selected linearization technique is tested on the IEEE 33-node distribution network system.**

***Index Terms*—active distribution network, distributed generation, mixed integer linear programming, linearized system loss**

I. Introduction

The basic task of Distribution Network Planning (DNP) is to determine the location, capacity of the substation, feeders and Distributed Generations (DGs) at a minimum investment scheme, satisfying load growth demand and the increasing uncertainties brought by uncontrollable DGs in the modern distribution system [1], [2]. The presence of DGs in distribution network can improve the reliability, efficiency and security of the system [3]. However, the output of non-dispatchable DGs, such as wind turbine generation and photovoltaic, are very volatile. They could critically cause operation problems in terms of system voltage, power quality and stability, fault level, and protection coordination, etc. [4], [5]. The planning of distribution network is facing challenges imposed by DGs due to their characters of intermittency and variability.

In a passive network scheme, usually called Traditional Distribution Network (TDN), DGs are generally installed based on ‘fit and forget’ approach and operated with fixed power factors [6]. It means that the load growth forecast is given and there is no control on the output of DG units to determine the installation of new distribution system assets [7]. And the ‘last-in-first-out’ approach for DG units is commonly applied by distribution system operator due to above reasons. The concept of active management of DG units can solve these problems, which is a significant part of Active Network Management (ANM) schemes. The major ANM schemes include DG’s Power Factor Control (PFC), Coordinated Voltage Control (CVC) of On-Load-Tap-Changers (OLTCs), compensator reactive power control, voltage regulators, and Energy Curtailment (EC) [6], [8]. ANM schemes can help the existing distribution systems accommodate more DG capacity, which will enhance the utilization of the distribution network’s assets, delay or avoid the upgrades of the network. And the above advanced technologies have converted the distribution network from passive to active, which is called Active Distribution Network (ADN).

DNP is a complex Mixed Integer Nonlinear Programming (MINLP) problem, which contains determining the place and the capacity of DGs, feeder lines and substations. In recent years, a lot of papers and studies have been carried out to solve this problem. The DNP problem can be divided into static model [9] and multistage model [10]. In the static model, the DNP problem is solved in a single stage, while the network investments are determined over successive planning stages based on the different requirements of each stage in the multistage model [11], [12]. This paper presents a new model optimizing the capacities of DG and the optimal distribution line simultaneously on the premise of that the topology of the network and the location of DGs are fixed, while the increasing Electric Vehicles (EV) in distribution network are also concerned. Various linearization technologies are used to transfer the complex MINLP problem of ADN planning into Mixed Integer Linear Programming (MILP) problem. The reasons of applying MILP model are that mixed integer linear techniques are nowadays very mature compared with genetic algorithms, they are fast, robust, and are able to solve problems with up to millions of variables [13].

The rest of this paper is organized as follows: Section II presents the proposed planning model of ADN. A piecewise linearization technique and a linearization

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technique based on operating point are introduced and compared in Section III in order to transfer the MINLP planning model into the stand MILP model. The above model with selected linearization technique is applied to the IEEE 33-node distribution network and the results are presented in Section IV. Conclusions are drawn in Section V.

II. Planning Model of ADN

In this section, a formulation of the active distribution network planning problem is presented. The planning model, which assumes that the location of DGs and the topology of the network are fixed, optimizes the capacities of DG and distribution line by a cost/benefit analysis and the benefit is quantified by the reduction of the expected interruption cost. The objective function is to maximize the social welfare, or it can be equally described as minimizing the total cost. The optimization model is set up as follows:

A. Objective Function

The objective function for ADN planning:

$$\min TC = C_{DG} + C_L + C_{grid} + C_{loss} + C_{LOL} \tag{1}$$

where TC is the abbreviation of total cost, which means the total cost of the ADN planning.

1) DG construction, operation and maintenance costs

$$C_{DG} = \sum_{i=1}^{N_{DG}} (\beta \cdot C_{equ} + C_{ope} + C_{rep})\lambda_i \cdot S_{DGi} + \beta \cdot C_{ins} \tag{2}$$

where N_{DG} is the number of total DG nodes containing DGs; β is DG fixed investment annual average cost factor:

$$\beta = \frac{r(1+r)^t}{(1+r)^t - 1} \tag{3}$$

where r is annual percentage rate, t is planning period. And C_{equ} is the equipment investment cost of DG on node i; C_{ope} is the operate cost of DG on node i; C_{rep} is annual maintenance cost of DG on node i; λ_i is power factor of *ith* DG unit; S_{DGi} is the rated capacity of DG on node i; C_{ins} is the fixed installation cost of DG unit on node i [14].

2) Feeders investment costs

$$C_L = \sum_{b=1}^{Nl} k_b \cdot l_b \cdot T_b \tag{4}$$

where b is the number of feeder line; N_l is the number of total distribution feeder lines; k_b is line annual investment cost of per unit length; l_b is line planning length; T_b is line optimal capacity.

In this model, several available line models selected for each feeder line are assumed as given data. For example, *LGJ-35*, *LGJ-50*, *LGJ-75*, *LGJ-120*, whose maximum allowable capacity is 2.9MW, 3.8MW, 5.6MW and 6.5MW, respectively. Each feeder line has this above four choices, while only one line model will be chose at last. A binary variable will be introduced in this model due to the above reasons.

3) Power purchase cost

$$C_{grid} = C_e(P_L - \sum_{i=1}^{n} \lambda_i \cdot S_{DGi}) \cdot T_{\max} \tag{5}$$

where C_e is the electricity price; P_L is the total load capacity of the distribution system; and T_{max} is the maximum load equivalent hours.

4) Power loss cost

$$C_{loss} = C_e \cdot P_{loss} \tag{6}$$

Several strategies to calculate the network loss P_{loss} will be introduced in detail in part 3.

5) Expected interruption cost

$$C_{LOL} = \sum_{i}^{N_i} \sum_{t}^{N_T} (\mathrm{VOLL} \cdot \mathrm{LOL}_{i,t}) \tag{7}$$

where N_i is the number of total nodes, N_T is the total time period, $LOL_{i,t}$ is the power expected interrupt loss on node i in time period t, and $VOLL$ is the value of lost load.

B. Constraints

$$\sum_{1}^{N_{DG}} P_{DG,t} + \sum_{1}^{N_{EV}} P_{EV,t} = \sum_{1}^{N_i} (P_{i,t} - LOL_{i,t}) + \sum_{1}^{N_i} P_{loss,i,t} \tag{8}$$

$$S_{ij} \le S_{ij}^{\max} \tag{9}$$

$$U_i^{\min} \le U_i \le U_i^{\max} \tag{10}$$

$$P_i^{G\min} \le P_{Gi} \le P_i^{G\max} \tag{11}$$

$$Q_i^{C\min} \le Q_i^C \le Q_i^{C\max} \tag{12}$$

$$T_k^{\min} \le T_k \le T_k^{\max} \tag{13}$$

$$P_{\min}^E \le P_t^E \le P_{\max}^E \tag{14}$$

$$C_{\min}^E \le C_t^E \le C_{\max}^E \tag{15}$$

$$C(0) = C_s,\ C(T) = C_{\max}^E \tag{16}$$

$$C(t+1) = C(t) - d_T P_{t,out}^E / \eta^E + d_T P_{t,in}^E \tag{17}$$

The constraints (8) enforce the total power balance, where N_{EV} is the number of electric vehicles. And the constraints (9) enforce line flow limits at every distribution line. The constraints (10) enforce the nodal voltage limits. In this model, the fluctuation range is within 7% of the normal operation voltage. The constraints (11) and (12) are output limits of units, where Q_i^c means the reactive power produced by a reactive compensator. And the constraints (13) is co-ordinated voltage regulation (area voltage control) using OLTC, T_k [15]. The constraints (14) ~ (17) are constraints that EV should satisfy [16], where P_t^E is the output power during period t; P_{max}^E is the maximum allowable charge/discharge limit; η_E is the discharging efficiency; T is the time duration of each period; C_t^E is the energy stored in EV until period t; $C(0)$ and $C(T)$ are starting and ending energy; C_{max}^E and C_{min}^E are maximum and minimum allowable energy stored in EV to ensure the efficiency and battery life of EV.

III. Calculation Strategies of Transmission Power and Feeder Line Loss

The network loss is explicitly analyzed in this section. For normal operation, under the flat voltage assumption, the network loss is appropriately simplified as a quadratic function of difference of voltage phase angle [17]. That is, the power injection in the line (i, j) computed at bus i, $p_{ij}(\delta_i,\delta_j)$, and the power injection in the line (i, j) computed at bus j, $p_{ji}(\delta_i,\delta_j)$, are given by:

$$P_{ij}(\delta_i,\delta_j)=U_i^2G_{ij}-U_iU_j(G_{ij}\cos\delta_{ij}+B_{ij}\sin\delta_{ij}) \quad (18)$$

$$P_{ji}(\delta_i,\delta_j)=U_j^2G_{ij}-U_iU_j(G_{ij}\cos\delta_{ij}-B_{ij}\sin\delta_{ij}) \quad (19)$$

where $y_{ij}=G_{ij}+jB_{ij}$, y_{ij} is the admittance of the line (i, j). We assume that $U_{ij}\approx 1$ in the normal operation of distribution network, then the power loss of the line (i, j) can be obtained as follows:

$$P_{loss}=P_{ij}+P_{ji}=2G_{ij}(1-\cos\delta_{ij})=G_{ij}(\delta_i-\delta_j)^2 \quad (20)$$

Then the power flow can be expressed as follows by using the above piecewise linearization methods:

$$P_{ij}=G_{ij}\left(\delta_i-\delta_j\right)^2-B_{ij}\sin\left(\delta_i,\delta_j\right)\cong\frac{1}{2}P_{ij}^{loss}\left(\delta_i,\delta_j\right)-B_{ij}(\delta_i-\delta_j) \quad (21)$$

$$P_{ji}=G_{ij}\left(\delta_i-\delta_j\right)^2+B_{ij}\sin\left(\delta_i,\delta_j\right)\cong\frac{1}{2}P_{ij}^{loss}\left(\delta_i,\delta_j\right)+B_{ij}(\delta_i-\delta_j) \quad (22)$$

where G_{ij} and B_{ij} are uncertain variable of the line (i, j). There are some binary variables and continuous variables included in the above equations so that the solving model is a MINLP problem. The crux of the problem lies in how to calculate the network loss, some different strategies are introduced in the following.

A. Traditional Piecewise Linearization Strategy

The feeder line loss in equation (20) can be piecewise linearized by using 2L piecewise linear blocks as shown in Fig. 1. The maximum range of values of $(\delta_i-\delta_j)$ is manually assigned as π/10, because the difference of voltage phase angle is usually small in reality and π/10 is enough. The value of this parameter will influence the calculation efficiency.

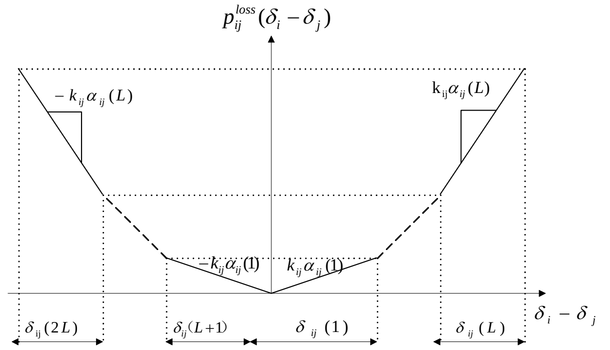

Figure 1. Piecewise linearization of network loss in a branch.

However, only L piecewise linear segements are sufficient by using the positive orthant only. In order to achieve this purpose, the linearization of absolute sign need to be introduced:

$$\delta_{ij}=\left|\delta_i-\delta_j\right| \quad (23)$$

$$\delta ij=\sum_{l=1}^{L}\delta_{ij}(l) \quad (24)$$

$$P_{ij}^{loss}(\delta_i,\delta_j)=G_{ij}\sum_{l=1}^{L}k_{ij}(l)\delta_{ij}(l) \quad (25)$$

where $k_{ij}(l)$ and $\delta_{ij}(l)$ means, the slope and value of the l th block of angle, respectively. The quadratic formulation of (20) is piecewise linearized to the above expression with the introduction of absolute sign. While the absolute value is still not a linear expression, a linear expression of the absolute value in (23) is needed, which is obtained by the following math substitution [18]:

$$\delta_{ij}=\delta_{ij}^{+}+\delta_{ij}^{-} \quad (26)$$

$$\delta_i-\delta_j=\delta_{ij}^{+}-\delta_{ij}^{-} \quad (27)$$

$$\delta_{ij}^{+}\geq 0,\delta_{ij}^{-}\geq 0 \quad (28)$$

Then the entire model can be transferred into a mixed integer linear model and then it can be solved by state-of-art MILP commercial solver such as CPLEX.

B. Piecewise Linearization Strategy Based on Preconditioning

Feeder line loss account for little proportion of network power in the distribution network. Its proportion is usually at around 5%, so that the feeder line loss will not affect the power injection and the difference of voltage phase angle seriously. A preconditioning technology based on the above premise can be introduced, and this approximate model without considering the feeder line loss can be expressed as following:

$$\min \mathrm{TC}=\mathrm{C_{DG}}+\mathrm{C_L}+\mathrm{C_{grid}}+\mathrm{C_{LOL}} \quad (29)$$

$$\sum_{1}^{\mathrm{N_{DG}}}\mathrm{P_{DG,t}}+\sum_{1}^{\mathrm{N_{EV}}}\mathrm{P_{EV,t}}=\sum_{1}^{\mathrm{N_i}}\left(\mathrm{P_{i,t}}-\mathrm{LOL_{i,t}}\right) \quad (30)$$

$$\mathrm{P_{ij}}=\mathrm{G_{ij}}\left(\delta_i-\delta_j\right)^2-\mathrm{B_{ij}}\sin\left(\delta_i,\delta_j\right)\cong-\mathrm{B_{ij}}(\delta_i-\delta_j) \quad (31)$$

$$\mathrm{P_{ji}}=\mathrm{G_{ij}}\left(\delta_i-\delta_j\right)^2+\mathrm{B_{ij}}\sin\left(\delta_i,\delta_j\right)\cong\mathrm{B_{ij}}(\delta_i-\delta_j) \quad (32)$$

The other equations are same as the model considering line loss. This preconditioning model is a simple MILP problem and can be solved efficiently by commercial solver CPLEX. Then we get the planning results including the difference of voltage phase angle δ_{ij0} of each line. After that, δ_{ij0} can be used in the main model to accelerate the calculation speed greatly based on the premise that δ_{ij} in the model considering line loss is approximately equal to δ_{ij0}:

$$0.8\delta_{ij0} \leq \delta_{ij} \leq 1.2\delta_{ij0} \tag{33}$$

C. *Linearization Strategy Based on Preconditioning and Operation Point*

Preconditioning model mentioned on Section 3.2 is also used in this strategy in order to get δ_{ij0} without considering network loss. The operation point of δ_{ij} is approximately equal to δ_{ij0}, satisfying the constraints (32), so that a linearization strategy based on this operation point can be put forward as following, and it is presented in Fig. 2.

$$k_{ij}(\delta_{ij0}) = 2 \cdot G_{ij} \cdot \delta_{ij0} \tag{34}$$

$$P_{ij}^{loss}(\delta_i, \delta_j) = k_{ij}(\delta_{ij0}) \cdot (\delta_{ij} - \tfrac{1}{2}\delta_{ij0}) \tag{35}$$

$$0.8\delta_{ij0} \leq \delta_{ij} \leq 1.2\delta_{ij0} \tag{36}$$

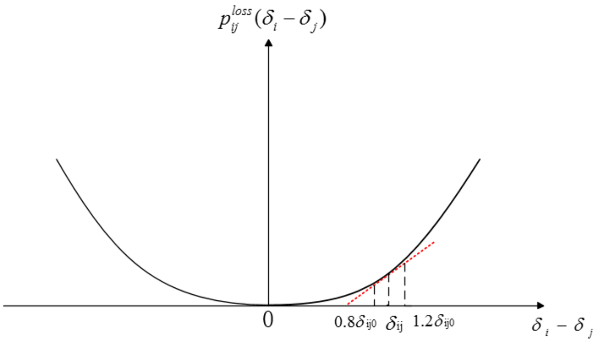

Figure 2. Linearization of network loss based on operation point.

IV. Case Study

The proposed model is tested on IEEE-33 node distribution test system and is coded in GAMS. The IEEE 33-node system used in this paper has 33 nodes, 32 loads, and 37 feeder lines concluding 5 tie lines, it is presented in Fig. 3.

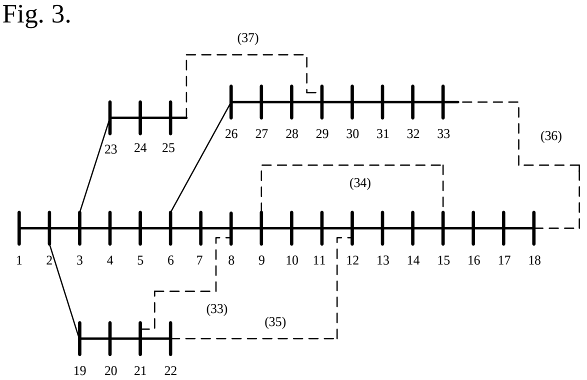

Figure 3. IEEE 33-node distribution system.

Table I shows the feeder line loss and calculation time by using different solving strategies mentioned in *Section 3*. Strategy 1 is the traditional piecewise linearization strategy which divides the value of δ_{ij} into several equational segments; strategy 2 is the piecewise linearization strategy based on preconditioning mentioned in Section 3.2; strategy 3 is the linearization strategy based on preconditioning and operation point introduced in Section 3.3, and strategy 4 is modeled as the original MINLP problem, and it is solved by using MINLP commercial solver BARON in GAMS.

TABLE I. Line Loss Results by Using Different Strategies

Strategy 1		
Number of segments	Line loss (kW)	Calculate time (s)
1	1442.300	15.6
2	729.972	26.2
3	493.468	35.4
5	309.518	162.6
10	157.728	127.9
15 20 25 30	Not converge	
Strategy 2		
Number of segments	Line loss (kW)	Calculate time (s)
1	37.466	7.9
2	32.817	9.0
3	29.880	10.1
5	29.844	12.5
10	30.634	18.0
15	30.310	24.3
20	29.619	30.5
25	29.619	47.9
30	29.619	92.2
Strategy 3		
Line loss (kW)	Calculate time (s)	
29.781	7.0	
MINLP		
Line loss (kW)	Calculate time (s)	
30.488	8.7	

It can be seen that the linearization strategy has great effect on the approximation accuracy and computational efficiency. For strategy 1, the approximation error is very considerable compared to other strategies when less linearization segments are used, but if more linearization segments are applied, the computation burden is heavier and the model will even become not converge when the number of segments is more than 10. Strategy 2 has reduced this problem to a certain degree by using less binaries and reducing the range of variables, the corresponding figure is shown in Fig. 4. It can be seen that the approximation result tends to a stable value with the linearization segments arises.

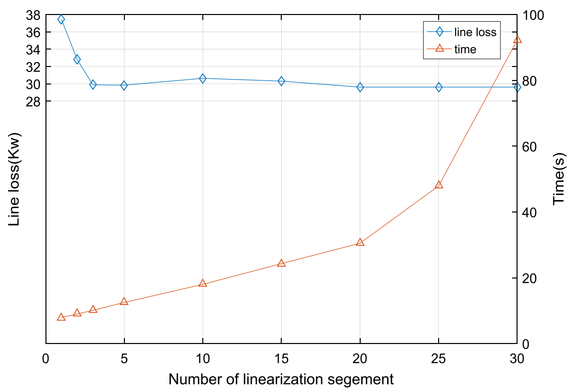

Figure 4. Variation of feeder line loss versus the number of segment.

However, when the scale of the distribute system becomes larger, or the planning model becomes multistage and dynamic, the computation burden by using strategy 2 will also become so heavy that it brings great barrier to model calculation. Strategy 3 does not have these above shortcomings. It has better approximation accuracy and computational efficiency compared with strategy 2, and the number of segments has no effect on the computation burden in this model by using a different linearization method. Finally, the strategy 3 is applied to the ADN planning model.

The optimal planning results of the capacities of DG and distribution line is showed in Table II and Table III. It should be noted that the node 1 here is a substation, which means the capacity showed on node 1 is the power purchased from upstream grid. And the model assumes that the installation location of the DG is given, here for the nodes 5,10,20,30, and then we only need to determine their capacities.

TABLE II. DG CAPACITY PLANNING RESULTS

Node number	1	5	10	20	30
Capacity(MW)	4.51	1.00	0	0	0

The distribution feeder line capacity planning results are as follows:

TABLE III. FEEDER LINE CAPACITY PLANNING RESULTS

Line index	Capacity (MW)	Line index	Capacity (MW)
1	5.6	20	2.9
2	5.6	21	2.9
3	5.6	22	2.9
4	5.6	23	2.9
5	5.6	24	2.9
6	2.9	25	2.9
7	2.9	26	2.9
8	2.9	27	2.9
9	2.9	28	2.9
10	2.9	29	2.9
11	2.9	30	2.9
12	2.9	31	2.9
13	2.9	32	2.9
14	2.9	33	2.9
15	2.9	34	2.9
16	2.9	35	2.9
17	2.9	36	2.9
18	2.9	37	2.9
19	2.9		

V. CONCLUSIONS

ADN planning is a complex MINLP problem. The model in this paper optimizes the capacity of DGs and distribution feeder lines simultaneously by a cost/benefit analysis. The network loss is explicitly analyzed and compared in this paper by using four different solving strategies, and strategy 3 is applied to the ADN planning model. Then the model can be transformed into a MILP problem so that it can be solved by state-of-art MILP commercial solver such as CPLEX. The effectiveness of the proposed ADN planning model with elaborate linearization technique is tested on the IEEE 33-node distribution network. The results show that the proposed linearization technique has better approximation accuracy and computational efficiency compared with other linearization methods.

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