Speed Control of a Variable Loaded DC Motor by Using Sliding Mode and Iterative Learning Control

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Abstract—In this paper, speed control of a DC Motor with time varying loaded is performed by using sliding mode control (SMC), classical PID control and iterative learning control (ILC) methods. SMC is a robust nonlinear control method which has insensitivity against to external disturbing effects and parametric variations of system. On the other hand, a control method of ILC provides an excellent performance on tracking. In the iterative learning PID (IL-PID) controller, the parameters of PID are automatically adjusted by using the algorithm of iterative learning. In this study, firstly, a DC Motor is modeled by using real data. Secondly, controllers which are an iterative learning PID (IL-PID), SMC-based and classical PID are designed and tested. Moreover, performance analysis of these controllers is done for load changes in the time interval. According to obtained results, the output of SMC-based system converges quickly to the reference value and the system gives the fastest response when changing of load occurs. Another result of this study is that the steady state error based on the learning success of ILC is decreased by IL-PID controller. The novel part of this study is that the comparison of these types of controllers is firstly made with this study.

Index Terms—control of motor, DC motor, iterative learning control, nonlinear control method, robust control, sliding mode control

I. INTRODUCTION

Many of the mechanical movement around us are provided by electric motors. Electric motors are devices that convert electrical energy to mechanical energy. Several explorers made the first motors supplied by direct current (DC) at the end of 1800s. DC motors are widely used in various industrial areas such as electric cars, steel raw material factories, electric cranes, robotic manipulators, paper machines and home applications because of their high efficiency, successful performance, high power density, large torque according to inertia ratio, certain, simple and steady control characteristics [1]-[3].

PID controllers are commonly used in DC motor applications and industries due to their simplicity structure. Although these controllers give good results in the control of LTI (Linear Time-Invariant) and easily modeled systems, they are not sufficient for time-varying, nonlinear and poorly modeled systems [4], [5]. In many applications, the parameters of DC motor could be changed by loading changes, temperature effect, current and voltage fluctuations [4]. The loading of DC motor generates fluctuations in motor speed [1].

PID controllers used in some applications of speed control are negatively affected by the loading due to parametric changes depending on motor’s location. Thus, the usage of these controllers is restricted. Until now, modified PIDs (adaptive, self-tuning etc.) have been developed to cope with changing-effects of the parameters, because fixed coefficients of PIDs exhibit a good performance [4].

In the study of [6], control of a PMDC motor is performed by using PI and fuzzy logic in simulation environment. Grouthaman et al. [7] make a study for speed control of PMDC motor via self-tuning PID control by using fuzzy logic. Velagic et al. [8] carry out speed control of a PMDC motor by way of fuzzy logic. Additionally, they make comparison for PID and fuzzy logic under disturbing effect by performing both simulation and real-time application. Kumar et al. [9] implement speed control of a separately excited dc motor via PI, PID and Ziegler Nichols methods. In the study of [10], PI controllers based on PSO, Ziegler Nichols and modified Ziegler Nichols are used for speed control of DC motor.

Classical methods such as Ziegler Nichols which are used to adjust parameters of PID controller can help for determining of these parameters just during design stage. However, they cannot be provided to online adjusting of controller parameters [11]. In addition, traditional PID controllers cannot provide steady state error of time varying systems to desired value. Therefore, iterative learning PID control (IL-PID) is proposed to eliminate deficiencies of traditional PID methods [12]. Iterative learning control (ILC) is used in industrial robots [13]-[15], DC motors [16], health care systems [17], [18] and many other areas. Moreover, it is considered that usage of SMC provides advantages to cope with mentioned negative effects.

In this study, analyzing for speed control of DC motor under load changes in certain time intervals is made by using iterative learning control and sliding mode control.
II. MODELING OF DC MOTOR

The equivalent circuit of DC motor to be used in this study is shown in Fig. 1 and speed control of the DC motor is provided by a fixed permanent magnetic field and adjusting armature voltage. The parameters used in model identification are explained in Table I.

![Schematic equivalent circuit of DC motor.](image)

Figure 1. Schematic equivalent circuit of DC motor.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Expression</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_a(t)$</td>
<td>Armature voltage</td>
<td>Volt</td>
</tr>
<tr>
<td>$i_a(t)$</td>
<td>Armature current</td>
<td>Amper</td>
</tr>
<tr>
<td>$e_b(t)$</td>
<td>Back emk</td>
<td>Volt</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Armature resistance</td>
<td>7.72 Ohm</td>
</tr>
<tr>
<td>$L_a$</td>
<td>Armature inductance</td>
<td>0.16273 Henry</td>
</tr>
<tr>
<td>$J$</td>
<td>Moment of inertia</td>
<td>0.0236 kg m²/s²</td>
</tr>
<tr>
<td>$B$</td>
<td>Viscous friction</td>
<td>0.003 Nms</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Torque coefficient</td>
<td>1.25 Nm/Amper</td>
</tr>
<tr>
<td>$K_b$</td>
<td>Back emk coefficient</td>
<td>1.25 V/srad</td>
</tr>
<tr>
<td>$T_L$</td>
<td>Load torque</td>
<td>Nm</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Torque produced by motor</td>
<td>Nm</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular velocity</td>
<td>Rad/s</td>
</tr>
</tbody>
</table>

The dynamic equations of DC motor are defined by equations given below:

\[
e_a(t) - e_b(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} \quad (1)
\]

\[
e_b(t) = K_b \omega(t) \quad (2)
\]

\[
i_a(t) = \frac{e_a(t)}{L_a} - \frac{K_b}{L_a} \omega(t) - \frac{R_a}{L_a} i_a(t) \quad (3)
\]

\[T(t) = T_L + J \frac{d\omega(t)}{dt} + B \omega(t) \quad (4)
\]

\[T(t) = K_t i_a(t) \quad (5)
\]

and firstly, Laplace transformation for these equations is performed under condition $T_I = 0$, later, the equations are synthesized, finally transfer function of DC motor is obtained as (6).

\[
e_a(s) = \frac{\omega(s)}{e_a(s)} = \frac{K_t J L_a}{\left[ s + \frac{R_a}{L_a} \right] \left[ s + \frac{B}{J} \right] + \frac{K_b K_t}{J L_a}}
\]

III. ITERATIVE LEARNING CONTROL

Iterative Learning Control (ILC) has a good tracking performance and it is an effective control method. When ILC is applied to a time-varying system, it improves transient state response and reduces trajectory error. The fundamental block diagram of ILC is shown in Fig. 2 [19].

![Block diagram of ILC.](image)

Figure 2. Block diagram of ILC.

where $u_k(t)$ is control signal for system, $y_k(t)$ is output signal, $y_d(t)$ is desired output signal and $u_{k+1}(t)$ is the control signal which is produced for next iteration. The basic expression of control signal for ILC system is defined by Arimoto et al. [20]. And it is given in (7) [21].

\[u_{k+1}(t) = u_k(t) + \Gamma \dot{e}_k(t) \quad (7)
\]

where $\Gamma$ is learning gain matrix. We can convert (7) to (8) which is similar to PID structure [20]:

\[u_{k+1}(t) = u_k(t) + \Phi e_k(t) + \Gamma \dot{e}_k(t) + \Psi \int e_k(t)dt \quad (8)
\]

This equation varies depending on the error and when it is applied to a system, the system can be optimized easily. Advantages of using ILC system are that it compensates disturbing effects to influence the system and makes the system more stable.

ILC method is also used in tuning PID controller parameters. PID parameters as (9), (10) and (11) according to structure of equation (8) can be defined [21].

\[K_{p_{t+1}} = K_p + \Phi e_t + \Gamma \dot{e}_t + \Psi \int e_t dt \quad (9)
\]

\[K_{i_{t+1}} = K_i + \Phi e_t + \Gamma \dot{e}_t + \Psi \int e_t dt \quad (10)
\]

\[K_{d_{t+1}} = K_d + \Phi e_t + \Gamma \dot{e}_t + \Psi \int e_t dt \quad (11)
\]

where for the first case $\Phi$, $\Gamma$ and $\Psi$ are proportional, derivative and integral constants, respectively. When these equations are examined, it is seen that PID parameters are tuned according to change of error. Design of block diagram for IL-PID controller is shown in Fig. 3.

In this system, steady-state error change both negative values and positive values. But this change causes increase and decrease of PID parameters randomly and this is contrary to fundamental principle of ILC. To solve this problem, absolute value of steady-state error must be used instead. In this way, proportional constant $K_p$, derivative constant $K_d$ and integral constant $K_i$ will
increase continuously. Then, we can define a saturated condition to prevent high value of PID parameters. For example, if the limit of error is set 0.001, the ILC system will produce PID parameters until the steady-state error reach this limit value and then PID parameters remain unchanged.

$$e(t) = y(t) - y_d(t)$$

If (13) is generalized as in (14), sliding surface can be shown as (15).

$$e(t) = [e_1(t), e_2(t), \ldots, e_n(t)]^T$$

Commonly used equation of sliding surface is as (16).

$$s(x, t) = \begin{bmatrix} d \end{bmatrix} + \lambda \begin{bmatrix} e(t) \end{bmatrix}$$

During the designing of sliding surface, \( s(x, t) = 0 \) is taken and homogenous differential equation whose solution is only \( e(t) = 0 \) is produced. Lyapunov function (17) is generally used for sliding mode and stability condition of system is obtained by using (18), which is shown as (19).

$$V = \frac{1}{2} s^2$$

While convergence condition is indicated as (20), sliding mode condition is shown as (21).

$$\dot{s} \leq -\eta |s|$$

System reaches to sliding mode for \( \eta > 0 \). If dynamic equations are progressed, \( u(t) \) control signal can be defined as in (22). This equation consists of two parts. The

IV. SLIDING MODE CONTROL

Sliding Mode Control (SMC), is a method based on Variable structure control (VSC). VSC is shown by Emelyanov et al. at the end of 1950s. On the other hand, SMC is unknown out of Russia until 1977 when Utkin does a scientific study [22]. It began to be used in international community after this study [23].

SMC is a robust control method to control complex and dynamic systems, which it has low sensitivity against to parametric variation, external effect. Also, it can operate under uncertain situations [22].

SMC consists of two parts as reaching mode and sliding mode and its phase portrait can be shown Fig. 4. First of all, sliding surface is designed for system to be controlled. Control signal which is switched high frequency slides the state trajectories of the system to this surface. In other words, SMC method tries to pull states of system from sliding surface to origin. The movement along this surface represents output action of the system. This movement in sliding surface represent sliding mode. On the other side, stage from starting point to sliding mode is called as reaching mode. If states of system are on sliding surface, it provides durability against to parametric variations and disturbing effects.

A. Basic Theory

Main purpose of SMC is that controlled system output \( y(t) \) can track desired output \( y_d(t) \) and \( u(t) \) which is highly minimized to tracking error can be produced. Thus, system reaches to sliding surface by passing between stable and unstable trajectories and error converges to zero in sliding surface.

Tracking error is defined as (12). Generalized form of (12) can be represented by (13).

$$e(t) = y(t) - y_d(t)$$

If (13) is generalized as in (14), sliding surface can be shown as (15).

$$e(t) = [e_1(t), e_2(t), \ldots, e_n(t)]^T$$

Commonly used equation of sliding surface is as (16).

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During the designing of sliding surface, \( s(x, t) = 0 \) is taken and homogenous differential equation whose solution is only \( e(t) = 0 \) is produced. Lyapunov function (17) is generally used for sliding mode and stability condition of system is obtained by using (18), which is shown as (19).

$$V = \frac{1}{2} s^2$$

While convergence condition is indicated as (20), sliding mode condition is shown as (21).

$$\dot{s} \leq -\eta |s|$$

System reaches to sliding mode for \( \eta > 0 \). If dynamic equations are progressed, \( u(t) \) control signal can be defined as in (22). This equation consists of two parts. The
first part of equation represents equivalent control signal, other part represents switching control signal whose equation is demonstrated as (23), where K is a positive value and sgn(·) is the signum function defined as (24).

\[
\begin{align*}
   u(t) &= u_{eq}(t) + u_{sv}(t) \quad (22) \\
   u_{sv}(t) &= K \text{sgn}(s) \quad (23) \\
   \text{sgn}(s(t)) &= \begin{cases} 
   +1, & \text{if } s(t) > 0 \\
   0, & \text{if } s(t) = 0 \\
   -1, & \text{if } s(t) < 0 
   \end{cases} \quad (24)
\end{align*}
\]

If \( s(t) \neq 0 \), switching control signal draws states of the system to sliding surface. After sliding surface is reached, it is disabled and stability of system is maintained via equivalent control signal.

Arisng from high frequency switching, chattering is a negative side of SMC method. Different studies can be performed to overcome this drawback.

B. Implementation of SMC

If simplifications and definitions in (25) are made by using (6), (26) can be obtained.

\[
M_1 = \frac{K}{JL_a} \quad M_2 = \frac{R_a}{JL_a} \quad M_3 = \frac{B}{J} \quad M_4 = \frac{K_b K_m}{JL_a} \quad (25)
\]

\[
\dot{\omega} + (M_2 + M_3) \dot{\omega} + (M_2 M_3 + M_4) \omega = e_\omega M_1 \quad (26)
\]

When descriptions for SMC are done as like (27), (28) is acquired.

\[
\begin{align*}
   x_1 &= \omega(t) \\
   y &= x_1 = \omega(t) \\
   x_2 &= \dot{x}_1 = \dot{\omega}(t) \\
   u &= e_\omega(t) \\
   \dot{x}_2 &= \dot{x}_1 = \dot{\omega}(t) \\
   X_2 + (M_2 + M_3)X_2 + (M_2 M_3 + M_4)X_1 &= M_1 u \quad (28)
\end{align*}
\]

Description for sliding surface can be made as follows (29) and definition of derivative of sliding surface is shown in (30).

\[
\begin{align*}
   s &= Cx_1 + x_2 \\
   s &= Ce + \dot{e} = C(\omega_r - \omega) + \omega_r - \dot{\omega} \quad (29) \\
   \dot{s} &= C(\ddot{\omega}_r - \ddot{\omega}) + \dot{\omega}_r + \dot{\omega} = 0 \\
   \dot{s} &= C(-\dot{\omega} - \dot{\omega}) + \dot{\omega} = 0 \\
   \dot{s} &= C(-\dot{\omega} - \dot{\omega} - K \text{sgn}(s)) \\
\end{align*}
\]

\[
\frac{1}{M_1} [(M_2 + M_3 - C) \dot{\omega} + (M_2 M_3 + M_4) \omega + K \text{sgn}(s)] = u \quad (31)
\]

(28) is substituted to (29) and parametric data of motor in Table I is replaced to (31) [24]. Later, if they are synthesized, \( u(t) \) control signal is produced as (32).

\[
M_1 = 325.484 \quad M_2 = 47.44 \quad M_3 = 0.127 \quad M_4 = 406.855
\]

\[
\frac{1}{325.484} [(47.567 - C) X_2 + (412.88) X_1 + K \text{sgn}(s)] = u \quad (32)
\]

For reducing of processing load and relieving of chattering, a change in function is made as like (33).

\[
\frac{1}{325.484} [(47.567 - C) X_2 + (412.88) X_1 + K \frac{s}{|s| + \delta}] = u \quad (33)
\]

0 < \delta < 1

Block diagram of sliding mode control is indicated in Fig. 5.

V. SIMULATIONS RESULTS AND COMPARISONS

In this section, comparisons for speed control performed by SMC, IL-PID and PID methods are made. Block schema of DC Motor is shown in Fig. 6.

For all controllers, graphs of speed control of DC Motor are given in Fig. 7 and Fig. 8. Firstly, when used PID control, %7.7 overshoot occurs and settling time is substantially increasing according to other controllers. Secondly, IL-PID is implemented to system. Then, it is seen that overshoot is decreasing and convergence to reference speed become earlier. Lastly, system is analyzed by using SMC. Moreover, it is observed that no overshoot and more quickly convergence is taken place according to other controllers. Thus, it can be said that SMC has shown a better performance.

0.1Nm load torque is integrated to motor system as a disturbing effect after 2.sec. In terms of transient, steady state analyzes and loading conditions, comparative speed-time graphs for PID, IL-PID and SMC are shown in Fig. 9 and Fig. 10. According to these graphs, SMC is more successful in point of reaching reference value and less overshoot while loading. As shown in Fig. 10, response of PID controller is too late and variation in speed is large. When the analysis is continued, if amount of loading is increasing or variation in loading occurs, it can be seen to
increase the performance difference. Graph drawn error and derivative of error for SMC is displayed in Fig. 11. In this figure, SMC pulls successfully error to origin. When disturbing effect happens, a small loop arises, and then error converges to zero. Convergence of sliding surface to zero is demonstrated in Fig. 12.

![Figure 7. Change of angular velocity vs. time for SMC, IL-PID and PID controller.](image)

![Figure 8. When zoomed, change of angular velocity vs. time for SMC, IL-PID and PID controller.](image)

![Figure 9. Change of angular velocity vs. time for SMC, IL-PID and PID controller (0.1 Nm load is integrated at 2 sec).](image)

![Figure 10. When zoomed, change of angular velocity vs. time for SMC, IL-PID and PID controller (0.1 Nm load is integrated at 2 sec).](image)

Figure 11. Phase portrait which is drawn error vs. derivative of error.

![Figure 12. Change of sliding surface for SMC.](image)

Performance comparison of controllers is given in Table II. It is explained by considering IAE, ISE and ITAE which are commonly used measurements of error.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Expression</th>
<th>PID</th>
<th>IL-PID</th>
<th>SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rising Time (s)</td>
<td>$T_r$</td>
<td>0.076</td>
<td>0.0574</td>
<td>0.045</td>
</tr>
<tr>
<td>Settling Time (s)</td>
<td>$T_s$</td>
<td>0.32</td>
<td>0.17</td>
<td>0.14</td>
</tr>
<tr>
<td>Overshoot (%)</td>
<td></td>
<td>7.7</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>Steady state error</td>
<td>$e_{SS}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Overshoot in loading (%)</td>
<td></td>
<td>-16.8</td>
<td>-2.74</td>
<td>-1.81</td>
</tr>
<tr>
<td>Time difference for settling in loading</td>
<td>$\Delta t$</td>
<td>0.45</td>
<td>0.45</td>
<td>0.05</td>
</tr>
<tr>
<td>Steady state error in loading</td>
<td>$e_{SS}$</td>
<td>0</td>
<td>0</td>
<td>0.0011</td>
</tr>
<tr>
<td>Integral Square Error</td>
<td>ISE</td>
<td>0.04015</td>
<td>0.02761</td>
<td>0.02398</td>
</tr>
<tr>
<td>Integral Absolute Error</td>
<td>IAE</td>
<td>0.06309</td>
<td>0.04015</td>
<td>0.03402</td>
</tr>
<tr>
<td>Integral Time Absolute Error</td>
<td>ITAE</td>
<td>0.003324</td>
<td>0.001087</td>
<td>0.0007477</td>
</tr>
</tbody>
</table>

VI. RESULTS

In this paper, performance of controllers designed by using SMC, IL-PID and PID methods for speed control of DC Motor that constitutes an important issue in control systems is compared. Also, an improvement is made to relieve chattering in SMC. At first, analyzes are made without any disturbing effect. Later, they are made according to time varying loading conditions and appreciations are carried out in point of criteria such as
settling time, steady state error and quick response to disturbing effect. It is analyzed that SMC method demonstrates successful recovery under especially loading. Moreover, SMC gives better results in transient analysis. However, in steady state, ILC operates less error due to learning structure of system. Using SMC method will be more advantageous in robotic applications where it is desirable that motors running under load should not be affected by external factors.

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REFERENCES


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