

# Health Status Evaluation of Low-Voltage Distribution Network Based on Rank Correlation-Entropy Method

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**Abstract**—In order to select bad health status of low-voltage distribution network to optimize, this paper assesses health status of low-voltage distribution network from 6 key indexes. Firstly establishes evaluation index system; secondly processes index consistently, and makes index values in [0, 1] with extreme value approach; then adds expertise with rank correlation analysis method to obtain subjective index weight, meanwhile obtains objective index weight with entropy method; finally acquires comprehensive index weight with the optimal Lagrange multiplier method, then assesses health status of low-voltage distribution network based on the evaluation function. Verify effectiveness and feasibility of evaluation methods through several low-voltage distribution networks.

**Index Terms**—low-voltage distribution network, health status evaluation, rank correlation analysis method, entropy method, the optimal Lagrange multiplier method

## I. INTRODUCTION

Distribution network has many features, including wide spread, many and random power lines, many types equipments, so there are many factors affecting health status of distribution network. Evaluation of distribution network has had many research, but the majority remain in the high and medium voltage distribution network, ref. [1] assesses the features of connection modes of high-voltage distribution network in reliability and economy; ref. [2] sets up evaluation system to adapt the characteristics of high-voltage distribution network planning; ref. [3] presents a method to accomplish the reliability assessment based on logical relation and matrix operations; ref. [4] evaluates status of urban high and medium voltage distribution network with AHP and Delphi method; ref. [5] proposes security assessment by Monte Carlo simulation with high penetration of wind power in modern distribution networks; ref. [6] investigates the use of symbolic dynamics to abstract raw data, and discusses algorithm using operators to calculate risk factors; ref. [7], [8] respectively process comprehensive evaluation from operational risk considering urgency [7], impacts of renewable energy and ICT driven energy transition on distribution networks

[8], so evaluation of high and medium voltage distribution network is mature and comprehensive.

Currently optimization and improvement of low-voltage distribution network are mainly based on user complaints, warning when station becomes abnormal operation, line running time. These have slightly effect on the results of optimization and improvement only by experience, lack appropriate analysis methods and decision-making tools, and give improper assessment for health status of low-voltage distribution network. So far there have had some achievements, including voltage characteristics, energy efficiency management, grid characteristics, load characteristics and other aspects for health status of low-voltage distribution network. Ref. [9] establishes evaluation index system from voltage characteristics, and sets evaluation index weight based on rank correlation-scatter degree method; ref. [10] establishes comprehensive evaluation index system of new rural low-voltage distribution networks in China based on Analytic Hierarchy Process (AHP); ref. [11] establishes index system from grid structure and load characteristics, and gets status assessment model based on improved rank correlation analysis method, the papers evaluate characteristic of low-voltage distribution network from different aspects, but lack health status evaluation.

How to efficiently select problem network to optimize and improve from so many low-voltage distribution network? Many factors effect health status of low-voltage distribution network, and health status assessment lacks reasonable assessment system, therefore this paper establishes evaluation index system from 6 key indexes in Section II, reflecting characteristics of health status, and process the 6 indexes with extreme value approach in Section III; uses rank correlation analysis method with expertise to obtain subjective weight, meanwhile utilizes entropy method with the difference of grid data to obtain objective weight, finally takes advantage of the optimal Lagrange multiplier method to obtain comprehensive weigh, making comprehensive weight as close to the objective and subjective, then gets evaluation function for health status of low-voltage distribution network, assesses health status by function value in Section IV, and provides decision-making foundation to optimize and improve low-voltage distribution network.

## II. EVALUATION INDEX SYSTEM

Considering health status of low-voltage distribution network and following evaluation index practicality, feasibility and easy to obtain in principle, uses 6 indexes to analyze health status of low-voltage distribution network. It is defined as follows.

*Power supply radius* ( $X_1$ ): It refers to power line length between user and transformers. It is an important parameter to assess health status of low-voltage distribution network. Long power supply radius leads to voltage problem, namely the longer the radius, the worse health status. Ideal power supply radius sequentially increases for urban, suburban and rural area.

*Economic deviation ratio of maximum diameter* ( $X_2$ ): It refers to deviation ratio between economic power line section area  $A_s$  on the low voltage side transformer and its actual section area  $A$ . Among  $A$  is the largest power line section area in low-voltage distribution network. The smaller  $X_2$ , the healthier status. It is calculated as follows.

$$X_2 = \frac{A_s - A}{A_s} = \frac{P_{\max} - \sqrt{3}\rho U_n \cos\varphi A}{P_{\max}} \quad (1)$$

where  $P_{\max}$  is the maximum transmission line active power,  $U_n$  is nominal line voltage,  $\rho$  is economic current density for line.

*Comprehensive line loss rate* ( $X_3$ ): It refers to ratio of line loss of power and total power supply of transformer in the low-voltage distribution network, reflecting whether distribution network works efficiently or not. The smaller comprehensive line loss rate, the healthier status. It is calculated as follows.

$$X_3 = \frac{P_1 - P_2}{P_1} \quad (2)$$

where  $P_1$  is line loss of power in the low-voltage distribution network,  $P_2$  is the total power supply of the transformer in the low-voltage distribution network.

*Transformer load rate* ( $X_4$ ): It refers to ratio of total active power supply of transformer and transformer rated active power, reflecting transformer load. Within the scope of transformer economic operation, the nearer middle of transformer load rate, the healthier status of low-voltage distribution network; when transformer is running below economic limit of range, indicating that the transformer capacity is too large, resulting in a waste of transformer; when transformer is running up the economic limit of range, indicating recent load growth is neglected, resulting in overload. It is calculated as follows.

$$X_4 = \frac{W}{ST} \quad (3)$$

where  $W$  is total active power supply of transformer in period  $T$ ,  $S$  is transformer capacity.

*User voltage qualification rate* ( $X_5$ ): It refers to ratio of number users number of voltage qualified and user number of the whole low-voltage distribution network, reflecting user satisfaction level. It is an important parameter to assess health status of low-voltage distribution network. The higher  $X_5$ , the healthier status.

*Three-Phase load unbalanced degree* ( $X_6$ ): It refers to maximum deviation ratio of transformer low-voltage side load and average load, reflecting unreasonable degree of load distribution among three phases. The smaller the degree of three-phase load unbalance, the healthier status of low-voltage distribution network. It is calculated as follows:

$$X_6 = \frac{3 \max\{P_A, P_B, P_C\} - (P_A + P_B + P_C)}{P_A + P_B + P_C} \quad (4)$$

where  $P_A, P_B, P_C$  are respectively transformer low voltage side  $A, B, C$  phase load.

## III. PROCESS INDEX

The health status indexes of low-voltage distribution network are divided into positive index, reverse index and range index, the bigger the benefit index value, the healthier status of low-voltage distribution network; the smaller the cost index value, the healthier; the closer to middle of index range, the healthier. Among health status index system, positive index: user voltage qualification rate ( $X_5$ ); reverse index: power supply radius ( $X_1$ ), economic deviation ratio of maximum diameter( $X_2$ ), comprehensive line loss rate ( $X_3$ ), three-phase load unbalanced degree ( $X_6$ ); range index: transformer load rate ( $X_4$ ). Dimension index includes  $X_1$ , other indexes are dimensionless.

Because amount of reverse index is more than others, consistently process every index into reverse index, namely the smaller the index value, the healthier status. After consistency processing, the index have different dimensions and types, so they aren't compared directly. Every index processes dimensionless based on extreme value approach, normalizing in  $[0, 1]$ . The following is three types of index processing formula.

It is positive index consistent and dimensionless processing formula as follows.

$$X_i^* = \frac{X_{i\max} - X_i}{X_{i\max} - X_{i\min}} \quad (5)$$

where  $X_{i\max}$  is the maximum index value,  $X_{i\min}$  is the minimum.

It is range index consistent and dimensionless processing formula as follows.

$$X_i^* = \frac{|X_{imid} - X_i|}{\max\{|X_{imid} - X_i|\}} \quad (6)$$

where  $X_{imid}$  is the middle of normal range index value.

It is reverse index dimensionless processing formula as follows:

$$X_i^* = \frac{X_i - X_{i\min}}{X_{i\max} - X_{i\min}} \quad (7)$$

## IV. ALGORITHM PROCESS

The health status evaluation method of low-voltage distribution network uses 6 evaluation indexes to assess the health status. In this paper, takes advantage of rank correlation analysis method, based on the principle of

function-driven, to judge the subjective index weight with expertise on importance of index, obtains subjective index weight; then uses entropy method, based on the principle of difference-driven in the data, to acquire objective index weight; then utilizes the optimal Lagrange multiplier method to get comprehensive index weight, closest to rank correlation analysis and entropy methods, finally obtains health status evaluation function, to calculate evaluation value and acquire assessment results.

A. Rank Correlation Analysis Method

Low-voltage distribution network needs to make various decisions, and focuses are different, so require that each index weight change flexibly with decision change, rank correlation analysis method meets the requirement, avoids so complex weight calculation process, has clear and simple process, meets need of health status evaluation, and reflects expert will. The process of rank correlation analysis is as follows.

1) Firstly determine order, the order relation for certain index evaluation criterion is  $Y_1 > Y_2 > \dots > Y_m$ .

2) Then determine degree of relative importance between adjacent indexes. Suppose that expert judges ratio of importance of evaluation index  $Y_{k-1}$  and  $Y_k$  as follows.

$$Y_{k-1}/Y_k = R_k \tag{8}$$

where  $k = m, m-1, \dots, 3, 2$ .

3) Finally subjective weight coefficient  $w_k$  is calculated as follows.

$$w_k = (1 + \prod_{k=2}^m \prod_{i=k}^m R_i)^{-1} \tag{9}$$

B. Entropy Method

In the light of information theory, information is a measure of system order degree, and entropy is a measure of system disorder degree. Entropy method is an objective weight method, based on the principle of difference-driven in the data, the bigger the difference between index, the smaller entropy, the greater index in the evaluation, the bigger weight. Entropy method principle steps as follows.

1) Process index

Firstly index  $X_{ij}$  ( $j$ -th index of  $i$ -th low-voltage distribution network) is processing, that the index proportion, in the same index, is processing. Calculate the proportion of the  $X_{ij}$ .

$$p_{ij} = \frac{X_{ij}}{\sum_{i=1}^n X_{ij}} \tag{10}$$

where  $i=1, 2, \dots, n$ .

2) Calculate entropy of  $j$ -th index

$$e_j = \frac{-1}{\ln n} \sum_{i=1}^n p_{ij} \ln p_{ij} \tag{11}$$

3) Calculate objective weight coefficient of  $j$ -th index

$$w_j = \frac{1 - e_j}{n - \sum_{j=1}^m e_j} \tag{12}$$

C. Comprehensive Weight

Rank correlation analysis method uses expertise to obtain index weight, only reflecting expert judgment, neglecting differences between index themselves to determine index weight; entropy method makes full use of difference in data to determine weight, only reflecting completely objective weight assignment, neglecting degree of index importance. Therefore it is the combination of rank correlation analysis method and entropy method that makes each index weight more reasonable and objective, not only reflects expertise, but also message of data itself.

Combine subjective weight  $w_i$  obtained by rank correlation analysis method with objective weight  $w_j$  obtained by entropy method, and acquire comprehensive weight  $W_i = \frac{w_i w_j}{\sum_{i=1}^n w_i w_j}$  ( $i = j$ ). According to the minimum

information entropy principle, building up

$$\min E = \sum_{i=1}^n W_i \left( \ln \frac{W_i}{w_i} \right) + \sum_{i=1}^n W_i \left( \ln \frac{W_i}{w_j} \right) \quad (i = j) \quad \text{to make}$$

comprehensive weight as close as possible subjective and objective weight, and use the optimal Lagrange multiplier method to optimize comprehensive weight, it is shown as follows.

$$W_i = \frac{(w_i w_j)^{\frac{1}{2}}}{\sum_{i=1}^n (w_i w_j)^{\frac{1}{2}}} \tag{13}$$

where  $\sum_{i=1}^n W_i = 1, W_i \geq 0, i=1, 2, \dots, n, i=j$ .

D. Health Status Evaluation Function

Health status evaluation function vector of low-voltage distribution network as follows.

$$\mathbf{Y} = \mathbf{X}\mathbf{W} \tag{14}$$

where  $Y_m$  is  $m$ -th health status evaluation function of low-voltage distribution network,  $\mathbf{Y} = (Y_1, Y_2, \dots, Y_m)^T$ .

$$\mathbf{X} = \begin{pmatrix} X_1(1) & \dots & X_n(1) \\ \vdots & & \vdots \\ X_1(m) & \dots & X_n(m) \end{pmatrix}$$

E. Evaluation Results Corresponding to Health Status Evaluation Function Value

Obtain health status evaluation function value by health status evaluation function, and refer to Table I to evaluate health status of low-voltage distribution network.

TABLE I. REFERENCE TABLE FOR HEALTH STATUS EVALUATION RESULT OF LOW-VOLTAGE DISTRIBUTION NETWORK

function value	[0, 0.3]	(0.3, 0.5]	(0.5, 0.7]	(0.7, 1]
status	great	good	average	bad

V. CASE ANALYSIS

In order to test the reasonableness of the above evaluation method, randomly select 8 low-voltage distribution networks within jurisdiction of a certain power supply bureau, and sequentially number network 1, 2, ..., 8.

Firstly collect data on the 8 low-voltage distribution network, including transformer model, load transformer load, the power line diameter of transformer low voltage side, user number, number of complaint users, user terminal voltage value, user load, wire economic current density and so on.

Sort out 6 original indexes of the 8 low-voltage distribution network, and process index according to (5), (6), (7). Index processed as shown in Table II.

TABLE II. PRETREATED INDEX

net.	pretreated index					
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>
n. 1	0.2058	0.8403	0.0046	0.0449	0.3614	0.4518
n. 2	0.0000	0.5573	0.0000	0.0635	0.3333	0.7267
n. 3	0.3911	0.0000	0.7526	0.1114	0.0000	0.5138
n. 4	0.0343	0.8893	0.6278	0.4374	0.8702	1.0000
n. 5	0.1389	0.3771	0.5128	0.0065	0.5454	0.2364
n. 6	1.0000	0.5866	0.4556	0.0818	0.4074	0.3612
n. 7	0.5060	0.9636	0.9684	1.0000	0.9118	0.5984
n. 8	0.7890	1.0000	1.0000	0.0021	0.6984	0.0000

TABLE III. SORTED INDEX IN RANK CORRELATION METHOD

original index	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>
sorted index	Y <sub>6</sub>	Y <sub>5</sub>	Y <sub>3</sub>	Y <sub>4</sub>	Y <sub>1</sub>	Y <sub>2</sub>

Secondly calculate the subjective index weight: assuming that expert judges each index importance in accordance with relevant information, the descending order of importance degree:  $Y_1 > Y_2 > Y_3 > Y_4 > Y_5 > Y_6$ , shown in Table III.

Assign  $R_k$  sorted by each index importance degree, then calculate subjective index weight according to (8), (9). Each index subjective weight is shown in Table V.

Thirdly calculate objective index weight: process index in Table II, namely use (10) to calculate the proportion of the  $j$ -th index of  $i$ -th low-voltage distribution network, as shown in Table IV; and calculate respectively entropy of each index and objective weight according to (11), (12). Each index objective weight is shown in Table V.

In calculating the entropy, when index is 0, (11), because of natural logarithm, emerges undetermined type  $\lim_{x \rightarrow 0} x \ln x = 0$ . It will not affect the results.

Fourthly calculate the comprehensive index weight: obtain comprehensive weight using (13), as shown in Table V, restore the original order.

TABLE IV. PRETREATED INDEX IN ENTROPY METHOD

net.	pretreated index in entropy method					
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>
n. 1	0.0671	0.1611	0.0011	0.0257	0.0888	0.1162
n. 2	0.0000	0.1069	0.0000	0.0363	0.0820	0.1869
n. 3	0.1276	0.0000	0.1741	0.0637	0.0000	0.1321
n. 4	0.0112	0.1706	0.1453	0.2503	0.1986	0.2572
n. 5	0.0453	0.0723	0.1187	0.0037	0.1341	0.0608
n. 6	0.3262	0.1125	0.1054	0.0468	0.1002	0.0929
n. 7	0.1651	0.1848	0.2241	0.5722	0.2244	0.1539
n. 8	0.2574	0.1918	0.2314	0.0012	0.1718	0.0000

TABLE V. SUBJECTIVE, OBJECTIVE AND COMPREHENSIVE WEIGHT

index	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>
sub.	0.0884	0.1149	0.1643	0.1264	0.2760	0.2300
obj.	0.1960	0.0816	0.1464	0.3855	0.0908	0.0977
com.	0.1140	0.1059	0.1697	0.2415	0.1732	0.1657

TABLE VI. HEALTH STATUS EVALUATION RESULT

net.	evaluation value	status
n. 1	0.2677	great
n. 2	0.2525	great
n. 3	0.2961	great
n. 4	0.6277	average
n. 5	0.2821	great
n. 6	0.4336	good
n. 7	0.8378	bad
n. 8	0.5107	average

Finally calculate health status evaluation value of low-voltage distribution network, and refer to Table I to evaluate health status as shown in Table VI.

According to the assessment result in Table VI, network 1, 2, 3, 5 are in great status, network 6 in good status, network 4, 8 in average status, network 7 in bad status. Analysis of network 7 poor state: Power supply radius is above normal range 95m, the diameter of line at transformer low voltage side is much smaller than the its economy diameter, the lowest voltage of user terminal is far less than 198V, line loss is more than half of the total power supply of transformer, users voltage qualification

rate is rarely, load during peak periods exceed the capacity of transformer.

Network 4 compared to network 7, although network 4 is more unbalanced three-phase load degree than network 7, but the network 4 other index are better than network 7.

Network 8 compared to network 7, transformer load rate, users voltage qualification rate and three-phase load unbalance degree in the network 8 is better than the network 7, others are slightly worse. But the network 8 is better than the network 7.

Index in network 1, 3, 5, 6 are basically reasonable, but power supply radius in the network 6 is too large, resulting in the network 6 in worst health status, the assessment result is good status.

Power supply radius and comprehensive line loss rate in the network 2 is optimal, three-phase load unbalance degree is slightly extent, other index are basically better than the same index.

The results show that the proposed method assesses health status of 8 low-voltage distribution networks, selects average and bad status network, where the network 7 is the worst and urgently needs to optimize and improve to change its health status; network 4, 8 are in average status, three-phase load unbalance degree in the network 4 is too large, so the network 4 needs to redistribute load to approach three-phase load balance; comprehensive line loss rate in the network 8 is excessive, and power supply radius is far beyond normal range, so network 8 need to reduce the line loss rate and power supply radius by increasing line diameter and optimizing its structure. The method selects problem low-voltage distribution network more efficiently and rationally, and proposes targeted optimization program.

## VI. CONCLUSIONS

(1) This paper proposes assessment method in health status of low-voltage distribution network, based on rank correlation analysis-entropy method. Establish index system from 6 main indexes. These indexes are easily obtainable, feasible, and able to efficiently and reasonably reflect the health status of low-voltage distribution network.

(2) Considering the 6 indexes characteristics and influences on health status of low-voltage distribution network, the 6 index are divided into positive index, reverse index and range index, then process index consistently using extreme value approach.

(3) The optimal Lagrange multiplier method makes comprehensive weight closest to subjective and objective, taking into account data characteristics and expertise, and avoiding defects of rank correlation analysis and entropy methods, so that comprehensive weight is more reasonable.

(4) Seen from the case, users voltage qualification rate, three-phase load unbalance degree, comprehensive line loss rate have great impact on health status of low-voltage distribution network. Therefore firstly select problem networks based on the three indexes. And then evaluate the networks, the bad network can be more efficiently selected, providing efficient and rational basis with

optimizing and improving low-voltage distribution network.

In summary, the proposed method is clear, concise, and suitable for health status evaluation of low-voltage distribution network, improving efficiency. Verify the validity and feasibility of evaluation methods by analyzing the case.

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