Robust Control Design of an Induction Motor

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***Abstract*—A robust controller for an AC induction motor is designed using principal gains method to govern stator currents. State space equations of an AC induction motor in α-β stator fixed frame are nonlinear with respect to rotor speed. Nonlinear equations are rewritten to parameter from describing dependency on rotor speed which is assumed to be known by measurement or by estimation. A nominal model (ω=0rd/s) is considered for synthesis, and all regimes dependent of ω are considered perturbed regimes at output multiplicative uncertainty. The singular values of uncertainty are quantified. Robust conditions of stability and performances are given. It is showed, from the results in frequency and time domain, that the principal gains method can be successfully applied to induction motor.**

***Index Terms*—principal gains, singular values, condition number, AC induction motor**

I. Introduction

The standard modern approach to induction motor control consists in constructing a mathematical model and than using explicitly this model in the controller. However, there are two major problems with this approach: first, the model is only a simplified representation of the dynamic AC induction motor which is generally more complex; second, the AC induction motor behavior continuously changes. For these two reasons there is inevitably a mismatch between the plant and the model. Such model uncertainties are responsible for the degradation of the controller. Hence, the first step in robust control study is to quantify these uncertainties. For that purpose, a simplified model of an AC induction motor (ω=0rd/s) and different models depending of the frequency are used to design a robust controller with principal gains method.

II. Preliminaries

It is necessary to recall the basic required performances of a control loop in frequency domain. Fig. 1 shows the classical structure of a control loop with the main components: the controller (transfer matrix K(s)), the process (transfer matrix G(s)), the multiplicative model uncertainty at the process output Δ_m(s), the set-point r, the loop's error e and finally the manipulated variable u and output y. let G'(s) the transfer matrix of the true plant, all perturbed regimes, then the following relation can be written:

$$G'(s) = [I + \Delta_m(s)]\, G(s) \tag{1}$$

The largest singular value of Δ_m(s) is obtained from "(1)":

$$\sigma_{max}\, [\Delta_m(s)] = \sigma_{max}\, ([G'(s) - G(s)]\, G^{-1}(s)) \tag{2}$$

"Equation (2) is used to quantify the multiplicative models uncertainties".

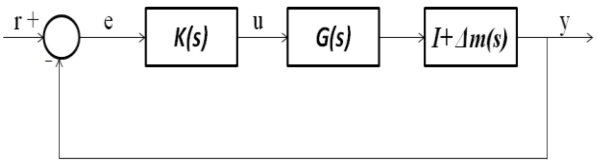

Figure 1. Feedback configuration with multiplicative uncertainties

A. Robust Stability

Assume that the nominal feedback system G(s) (i.e. with Δ_m(s) = 0) is stable, then the true feedback system G'(s) is stable if the following inequality holds [1]:

$$\sigma_{max}\, [T(s)] < 1/\, \sigma_{max}\, [W_t(s)] \tag{3}$$

where T(s) is the nominal closed loop transfer matrix given by:

$$T(s) = G(s)\, K(s)\, [I + G(s)\, K(s)]^{-1} \tag{4}$$

And W_t(s) is stability specification matrix such as:

$$\sigma_{max}\, [\Delta_m(s)] \le \sigma_{max}\, [W_t(s)] \tag{5}$$

Then the maximum principal gains σ_{max} [T(s)], the largest singular value of the nominal closed loop transfer matrix is a reliable indicator of the robust stability of the feedback system. "Equation (3) is the robustness condition of the feedback system".

B. Robust Performances

Let W_p (s) a performance specification matrix, weighting matrix, than the robust performances of all

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perturbed regimes G'(s) are satisfied if the following inequality holds [1], [2], [3]:

$$\sigma_{max}\ [S(s)] \leq 1/\ \sigma_{max}\ [W_p(s)] \tag{6}$$

where S(s) is the sensitivity matrix given by:

$$S(s) = [I + G(s)K(s)]^{-1} \tag{7}$$

In fact, the largest singular value of the sensitivity matrix σ_{max} (s) is also an indicator of the sensitivity of the system response to a change of the plant character.

In conclusion, the inequalities "(3)" and "(6)" represent the robustness conditions and must be satisfied to obtain a robust controller.

III. Principal Gains Method

The principal gains method is based on finding a controller with the following structure [3]:

$$K(s) = K1*K2(s)*K3*K4(s) \tag{8}$$

where:

K1= G^{-1} (0) is the inverse static gain. It is used to decouple the process in low frequency.

K2(s) = I/s is a set of integrators to eliminate the static error.

K3 is a compromise coefficient between the stability and performances.

K4(s) is a structure to reduce the resonance magnitude in middle and high frequency. In order to not affect the controller in low frequency, we have to set K4 (0) =I, this can be obtained by minimization of the following criteria [3], [4]:

$$min_{\ K4}(J) = min_{\ K4} max_{\ W}\ [\sigma_{max}(T)\ \sigma_{max}\ (\Delta_m)] \tag{9}$$

where: $\sigma_{max}(T)$, $\sigma_{max}(\Delta_m)$ is a stability robust condition.

IV. Application to the Ac Induction Motor

The state space equations of an AC induction motor dependent on ω= [-110rd/s, 110rd/s] are given in [5] and [6]:

$$\dot{x} = (A_0 + \omega A_1)\ x + Bu$$
$$y = Cx + Du \tag{10}$$

With state vector x = $[\varphi_a, \varphi_b, i_a, i_b]^T = [x_1, x_2, x_3, x_4]^T$, where φ_a, φ_b are the rotor fluxes and i_a, i_b are the stator currents and the control input u = $[u_1,u_2]^T$ represents the stator voltages. The measured output is y = $[i_a, i_b]^T$.

The matrices A_0, A_1, B, C and D are given as:

$$A_0 = \begin{pmatrix} a_1 & 0 & a_2 & 0 \\ 0 & a_1 & 0 & a_2 \\ a_3 & 0 & -\gamma & 0 \\ 0 & a_3 & 0 & -\gamma \end{pmatrix} \tag{11}$$

$$A_1 = \begin{pmatrix} 0 & -n_p & 0 & 0 \\ n_p & 0 & 0 & 0 \\ 0 & a_4 & 0 & 0 \\ -a_4 & 0 & 0 & 0 \end{pmatrix} \tag{12}$$

$$B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ a_5 & 0 \\ 0 & a_5 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{13}$$

where the following parameters are given in Table I:

$$a_1 = -\frac{1}{T_r},\ a_2 = \frac{L_{sr}}{T_r},\quad a_3 = \frac{L_{sr}}{T_r\sigma L_s L_r},\ a_4 = \frac{n_p L_{sr}}{\sigma L_s L_r},\quad a_5 = \frac{1}{\sigma L_s},$$

$$T_r = \frac{L_r}{R_r},\quad \gamma = \frac{R_s}{L_s\sigma} + \frac{L_{sr}^2}{L_s\sigma L_r T_r},\quad \sigma = 1 - \frac{L_{sr}^2}{L_s L_r}$$

TABLE I. The Nominal Physical Parameters of the Induction Motor

Description	Parameter	Value	Units
Stator Inductance	L_s	0.47	H
Rotor Inductance	L_r	0.47	H
Mutual Inductance	L_{sr}	0.44	H
Leakage factor $\sigma_s = \sigma_r$	σ	0.12	
Stator resistance	R_s	0.8	Ω
Rotor resistance	R_r	3.6	Ω
Moment of inertia	D_m	0.06	*Kg.m²*
Viscous damping constant	R_m	0.04	*N.m.s*
Number of pole pairs	n_p	2	

The step responses for -110 rd/s ≤ω≤ 110rd/s are represented in Fig. 2 and Fig. 3:

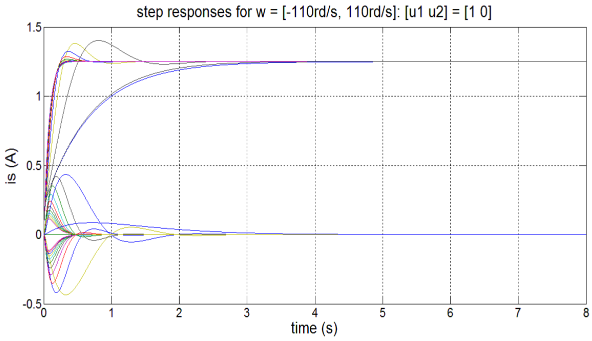

Figure 2. Open loop time responses for ω= [-110rd/s, 110rd/s]. Unit step demand in u_1

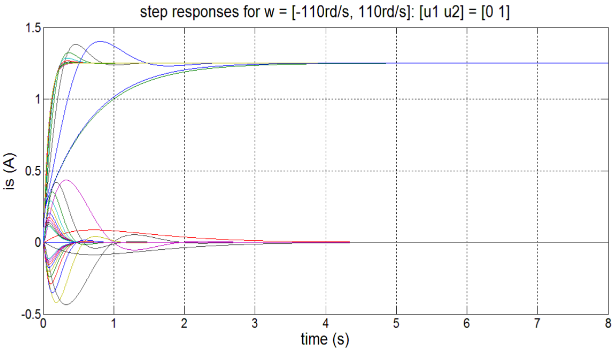

Figure 3. Open loop time responses for ω= [-110rd/s, 110rd/s]. Unit step demand in u_2

From the Fig. 2 and Fig. 3 it is clearly that there are cross-couplings and badly damped modes whose frequencies vary considerably with the rotor speed. This coupling is also showed in Fig. 4 because the condition number [7]-[9], is between 2 and 8.

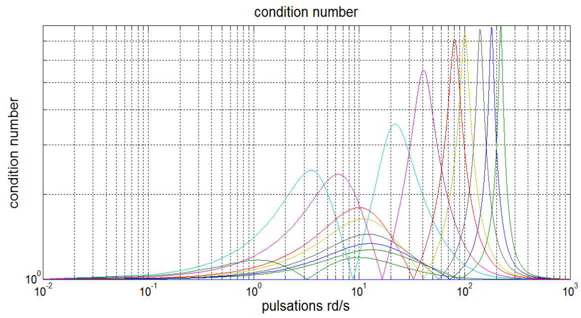

Figure 4. Condition number of induction motor (-110 rd/s ≤ ω ≤110rd/s)

A. Evaluation of Multiplicative Uncertainties $\Delta_m(s)$

The largest singular values of the multiplicative uncertainties Δ_m(s) are determined from "(2)".

The result is given in Fig. 5, where it is verified that the norms of these uncertainties are less than one at low frequencies and increase at high frequencies [1].

B. Robustness Conditions

Using "(5)" and the result given in Fig. 5, the stability specification matrix W_t(s) is represented as:

$$W_t(s) = \begin{pmatrix} 0.8(1+1.5s) & 0 \\ 0 & 0.8(1+1.5s) \end{pmatrix} \quad (14)$$

Then, the condition for robust stability is given by "(3)". The performance specification for all possible plants, perturbed regimes, are defined such these regimes have the same response time that the nominal regime when the pulsation ω = 0 rd/s, then the nominal dynamic matrix is A_0. Then, the performance specification matrix W_p(s) is given by:

$$W_p(s) = \begin{pmatrix} \frac{2s+1}{2s} & 0 \\ 0 & \frac{2s+1}{2s} \end{pmatrix} \quad (15)$$

The condition for robust performance is given by "(6)".

Finally, the robustness conditions for AC induction motor are represented in Fig. 6.

C. Robust Controller with Principal Gain Method for AC Induction Motor

The controller is given by "(8)" where:

$$K1 = G^{-1}(0) = \begin{pmatrix} 0.8 & 0 \\ 0 & 0.8 \end{pmatrix} \; ; \quad K2(s) = \frac{I}{s} = \begin{pmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{pmatrix}$$

K3 = 0.6 is determined by simulation.

K4(s) is obtained by minimization of the criteria "(9)":

$$K4(s) = \begin{pmatrix} 1+0.07261s & 0 \\ 0 & 1+0.07261s \end{pmatrix}$$

Finally the controller is:

$$K(s) = \begin{pmatrix} \frac{0.04066s+0.56}{s} & 0 \\ 0 & \frac{0.04066s+0.56}{s} \end{pmatrix} \quad (16)$$

The results in frequency domain are given in Fig. 7, where it is showed that the robustness condition are not violated because, for multivariable system, the stability is guaranteed if the largest singular values of the closed loop transfer matrix (σ_{max}[T(s)] is lower than the upper bound of the largest singular values of the model uncertainties (1/σ_{max} [W_t(s)]). The same idea is used for the robust performance criterion.

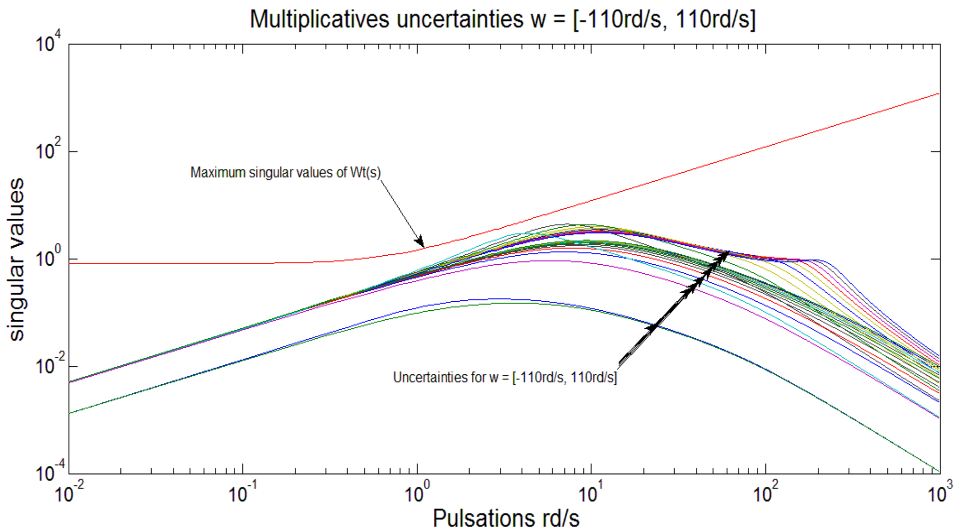

Figure 5. Multiplicative uncertainties Δm(s)

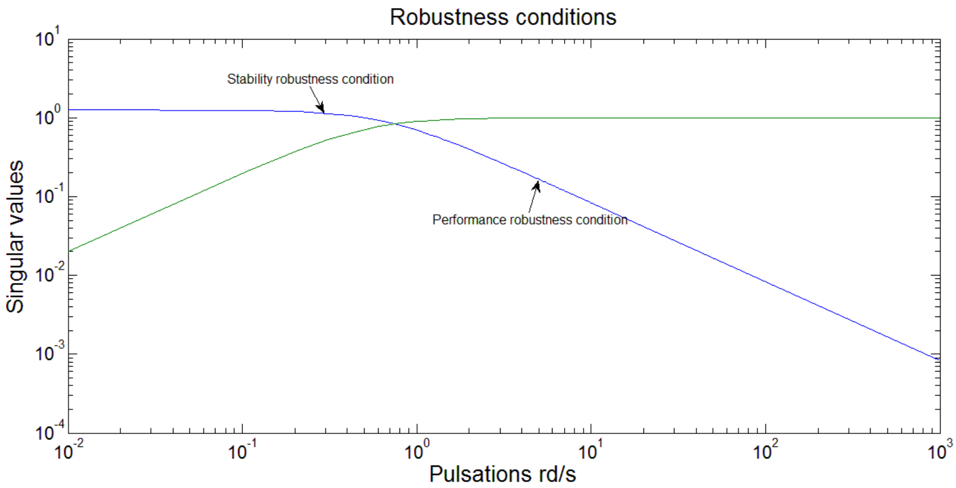

Figure 6. Robustness conditions of stability and performances.

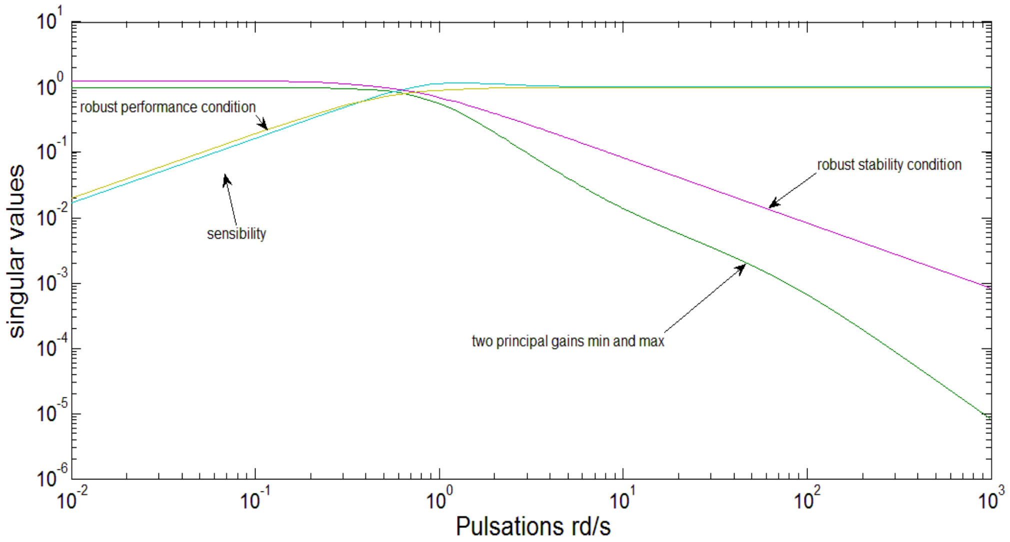

Figure 7. Results in frequency domain

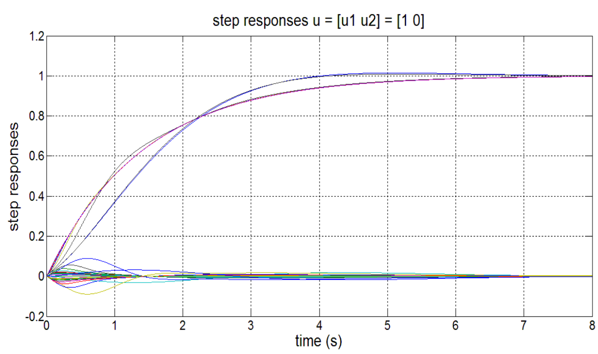

Figure 8. Step responses: echelon u1.

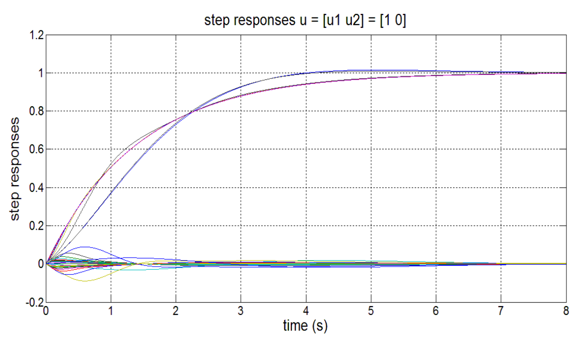

Figure 9. Step responses: echelon u2.

In Fig. 8 and Fig. 9 the results in temporal domain are given; the stability of all regimes and good performances which means small interactions, are observed.

V. CONCLUSION

This article presented principal gains method to design robust controller for an AC induction motor. It is showed, from the results in frequency and time domain, that the principal gains method can be successfully applied to induction motor. The dynamic behavior of induction motor is generally difficult to model, it could be done. The theory behind the robust control tools is simplified to be easily transmitted to electrical processing students and engineers.

REFERENCES

[1] J. C. Doyle and G. Stein, "Multivariable feedback design: Concepts for a classical modern synthesis," *IEEE Trans. Automat. Control,* vol. AC-26, no. 1, pp. 4-16, 1981.

[2] M. G. Safonov and R. Y. Chiang, "CACSD using the state space L_∞ theory-a design example," *IEEE Trans. Automat. Control,* vol. 33, no. 5, pp. 477-479, 1988.

[3] S. Yahmedi, "Design of algorithms tools for the study of stability and performances robustness of multi variables systems," PhD Thesis (French Text). Univ. Laval, Québec, Canada, 1993.

[4] Z. Q. Wang, K. D. Hu, and Z. H. Qian, "Design of a robust controller in frequency domain," presented at the IFAC 10th Tri-annual World Congress, Munich, FRG 1987.

[5] A. Benchaib and C. Edwards, "Non linear sliding mode control of an induction motor," *International Journal of Adaptive control and Signal Processing*, vol. 14, no. 2-3, pp. 201-221, 2000.

[6] E. Prempain, I. Postlethwaite, and A. Benchaib, "A linear parameter variant H_∞ control design for an induction motor," *Control Engineering Practice*, vol. 10, no. 6, pp. 633-644, 2002.

[7] S. Skogestad, M. Morari, and J. Doyle, "Robust control of ill-conditioned plants: High-purity distillation," *IEEE Trans. Automat. Control,* vol. 33, no. 12, pp. 1092-1105, 1988.

[8] S. Skogestad and M. Morari, "Some new properties of the structured singular value," *IEEE Trans. Automat. Control,* vol. 33, no. 12, pp. 1151-1154, 1988.

[9] M. Morari and E. Zafirion, *Robust Process Control*, Prentice Hall, Englewood Cliffs, NJ. 1989.

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