

# Effects of Unbalanced Voltage on the Steady State of the Induction Motors

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**Abstract**—This paper examines the proper application of three phase induction motors when supplied by unbalanced voltage. To analyze the effects of unbalanced voltage on torque of the induction motors, it has defined 32 unbalanced points, with 8 different types of voltage unbalance, and four levels of Voltage Unbalanced Factor, measured with Symmetrical Components method. In each unbalanced point the Torque Ripple Factor are calculated to show the effects of unbalanced voltage.

**Index Terms**—induction motors, unbalanced voltage, steady state

## I. INTRODUCTION

Due to the incomplete transposition of transmission lines, unbalanced loads, open delta transformer connections, blown fuses on three-phase capacitor banks and so on, lead to power systems unbalance putting worries for the power companies.

Even unproportional unbalance in the line currents are caused by a small unbalance in the line voltages. Because of various techno-economic benefits the induction motors are widely used in industrial, commercial and residential systems. Once the voltage is unbalanced, the ill effects on induction motors will cause enormous impacts. A lot of efforts have been reported in literature the adverse effects on induction motor performance under unbalanced voltage supply condition [1]-[7]. Over-voltage unbalance is defined as unbalance due to the positive-sequence voltage component being higher than the balanced rated voltage. Most of the research has been focused on voltage unbalance caused by under-voltage. While over-voltage cases often occur in the off-peak periods. Most of the regulations about voltage unbalance only specify the percentage of voltage unbalance factor (VUF) without indicating the unbalance conditions e.g., the British standard BS-4999 “General Requirements for Rotating Electrical Machines” [8] states that motors should deliver their rated horsepower when fed continuously from a supply having VUF of 2 percent or less, and the NEMA Standard MGI 12.45-1987, “Voltage Unbalance” [9] states that operation of a motor above 5 percent voltage unbalance is not recommended. But the reality is that there are many voltage unbalance cases which have the same voltage unbalance factor (VUF). If only VUF is

mentioned then neither the exact voltage unbalance situation can be estimated nor can the impacts on the power system due to voltage unbalance not be evaluated. In the last years some works has analyzed the influence of voltage unbalance on the steady-state performance of an induction motor, i.e.: [3], [4] and [6]. The voltage unbalance produce that an induction motor increase: losses, heating, noise, vibration, torsional pulsations, slip and motor accelerating torque [7].

Other works (i.e. [5] and [6]), studied the ripple in torque and speed to detect the unbalance supply voltage.

In [8] the analysis is focused on the comparison of the calculation method of phase and line voltage unbalance factor by load unbalance.

In [9] is made a statistical study with the efficiency and power factor of induction motor when are submitted a voltage unbalanced. These variables are related with positive and negative sequence of voltage, and Voltage Unbalance Factor.

In this work, the effects of voltage unbalance on torque of induction motors have been analyzed. For this task 32 voltage unbalanced points are been chosen, to study different levels and types of voltage unbalance, to observe the consequences on torque.

In Section II the types of voltage unbalance are presented. In Section III the selected method to calculate the voltage unbalance and the limits of the standards for this factor are presented in Section IV. The 32 unbalanced points are defined in Section V. The main characteristics of induction motor are presented in Section VI. In Section VII the procedure to make the simulations is defined. In Section VIII is showed the time evolution of torque of induction motor, when the voltage is unbalanced. In Sections IX and X the results for the effects of unbalance voltage on torque is presented. Finally the conclusions are in Section XI.

## II. TYPES OF VOLTAGE UNBALANCE

In [7] was showed a classification for voltage unbalance: 1: single phase under-voltage (1 $\Phi$ -UV); 2: two phases under-voltage (2 $\Phi$ -UV); 3: three phases under-voltage (3 $\Phi$ -UV); 4: single phase over-voltage (1 $\Phi$ -OV); 5: two phases over-voltage (2 $\Phi$ -OV); 6: three phases over-voltage (3 $\Phi$ -OV); 7: unequal single phase angle displacement (1 $\Phi$ -A); 8: unequal two phases angles displacement (2 $\Phi$ -A).

In the Table I the phasors for each type of voltage unbalance are presented, where each circle represents the value 1 pu for the voltage module.

In this paper are analyzed all voltage unbalance types.

TABLE I. PHASORS FOR THE EIGHT TYPES OF VOLTAGE UNBALANCE

		Phasors	Time evolution
Unbalanced type	T1 1Φ-UV		
	T2 2Φ-UV		
	T3 3Φ-UV		
	T4 1Φ-OV		
	T5 2Φ-OV		
	T6 3Φ-OV		
	T7 1Φ-A		
	T8 2Φ-A		

### III. VOLTAGE UNBALANCE FACTOR

In [10] is presented the Complex Voltage Unbalance Factor ( $VUF_{SC}$ ), as shown in (1).

$$VUF_{SC}(\%) = \frac{V_2}{V_1} 100 \quad (1)$$

In [2] and [8] are commented different methods to define the *Voltage Unbalance Factor*, but the negative-to-positive sequence modules ratio method is preferred because it directly represents the phenomena of interest without approximation [2]. This method as shown in (2).

$$VUF_{SC}(\%) = \frac{|V_2|}{|V_1|} 100 = \frac{V_2}{V_1} 100 \quad (2)$$

The expressions (1) and (2) can be used with phase-to-neutral voltages, phase-to-phase voltages and line currents.

Normally the equipment only measures the module of voltages, in this case the expressions (3) and (4) give other way to calculate the Voltage Unbalance Factor, but are only valid for the phase-to-phase voltages and line currents. In other words, these expressions can't be used with phase-to-neutral voltages

$$VUF_{\beta}(\%) = \sqrt{\frac{1 - \sqrt{3 - 6\beta}}{1 + \sqrt{3 - 6\beta}}} 100 \quad (3)$$

where:

$$\beta = \frac{|V_{ab}|^4 + |V_{bc}|^4 + |V_{ca}|^4}{(|V_{ab}|^2 + |V_{bc}|^2 + |V_{ca}|^2)} \quad (4)$$

In this paper the Voltage Unbalance Factor with Symmetrical Components method is selected.

### IV. LIMITS OF VOLTAGE UNBALANCED FACTOR

Most of the standards agree on limiting the voltage imbalance to 2 % [11]. For this reason, in this paper, four different levels of this factor, with Symmetrical Components method ( $VUF_{SC}$ ), has been taken: 1 %, 2 %, 3 % and 5 %.

Just the value  $VUF_{SC} = 2\%$  is the limit adopted. For values equal or lower of this, the results are inside of the limits of the standards.

With this study is possible see the influence of the four levels of Voltage Unbalanced Factor on torque and current of the induction motors.

### V. UNBALANCED POINTS

To analyze the effects of the unbalanced voltage on the torque and current of induction motors, 32 unbalanced points are taken, for the eight different types of voltage unbalance defined in Section II, and for the four levels defined in Section IV.

In the Table II are showed the values for the module and angle of the phase-to-neutral voltages taken in this work to obtain the voltage unbalance level and type desired.

The criteria follow for each type to arrive to the desired value of  $VUF_{SC}$  was: T1: the phase a has been selected to reduce the rms voltage; T2: phases b and c has been selected to reduce the rms voltage, and the same reduction was token for both; T3: the rms voltage of c phase has been reduced to 0.99 pu, the rms voltage of b phase is the average of rms voltages of a and c phases, and rms voltage of a was sought to obtain the desired level of  $VUF_{SC}$ ; T4: the phase a has been selected to increase the rms voltage; T5: phases b and c has been selected to increase the rms voltages, and the same increase was token for both; T6: the rms voltage of c phase has been increased to 1.01 pu, the rms voltage of b phase is the average of rms voltages of a and c phases; T7: the phase a has been selected to make a angle displacement; T8: the phases b and c has been selected to make the symmetrical angle displacement.

Other combinations are possible to arrive at the same conditions in each unbalanced point showed in Table II.

TABLE II. VALUES OF MODULE (PU) AND ANGLE OF THE PHASE-TO-NEUTRAL VOLTAGES FOR THE 32 UNBALANCED POINTS, IN FUNCTION OF UNBALANCE TYPE AND VOLTAGE UNBALANCE FACTOR.

		$VUF_{SC} = 1\%$	$VUF_{SC} = 2\%$	$VUF_{SC} = 3\%$	$VUF_{SC} = 5\%$
Unbalance type	T1	$V_{an} = 0.97030$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 0.94118$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 0.91263$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 0.85715$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$
	T2	$V_{an} = 1.00000$ $V_{bn} = 0.97059$ $V_{cn} = 0.97059$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.00000$ $V_{bn} = 0.94231$ $V_{cn} = 0.94231$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.00000$ $V_{bn} = 0.91510$ $V_{cn} = 0.91510$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.00000$ $V_{bn} = 0.86364$ $V_{cn} = 0.86364$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$
	T3	$V_{an} = 0.95629$ $V_{bn} = 0.97315$ $V_{cn} = 0.99000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 0.92371$ $V_{bn} = 0.95686$ $V_{cn} = 0.99000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 0.89220$ $V_{bn} = 0.94110$ $V_{cn} = 0.99000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 0.83219$ $V_{bn} = 0.91110$ $V_{cn} = 0.99000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$
	T4	$V_{an} = 1.03030$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.06121$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.09279$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.15790$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$
	T5	$V_{an} = 1.00000$ $V_{bn} = 1.03060$ $V_{cn} = 1.03060$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.00000$ $V_{bn} = 1.06250$ $V_{cn} = 1.06250$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.00000$ $V_{bn} = 1.09575$ $V_{cn} = 1.09575$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.00000$ $V_{bn} = 1.16665$ $V_{cn} = 1.16665$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$
	T6	$V_{an} = 1.04560$ $V_{bn} = 1.02780$ $V_{cn} = 1.01000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.08250$ $V_{bn} = 1.04630$ $V_{cn} = 1.01000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.12070$ $V_{bn} = 1.06540$ $V_{cn} = 1.01000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.20154$ $V_{bn} = 1.10577$ $V_{cn} = 1.01000$ $\alpha_a = 0^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$
	T7	$V_{an} = 1.00000$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 1.718^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.00000$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 3.437^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.00000$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 5.154^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$	$V_{an} = 1.00000$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 8.581^\circ$ $\alpha_b = -120^\circ$ $\alpha_c = 120^\circ$
	T8	$V_{an} = 1.00000$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -119.00^\circ$ $\alpha_c = 119.00^\circ$	$V_{an} = 1.00000$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -118.00^\circ$ $\alpha_c = 118.00^\circ$	$V_{an} = 1.00000$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -116.98^\circ$ $\alpha_c = 116.98^\circ$	$V_{an} = 1.00000$ $V_{bn} = 1.00000$ $V_{cn} = 1.00000$ $\alpha_a = 0^\circ$ $\alpha_b = -114.91^\circ$ $\alpha_c = 114.91^\circ$

VI. INDUCTION MOTOR

The induction motor selected to obtain the results presents the next topics: squirrel cage, 75 kW; 3300 V; 50 Hz; 15.32 A; 1455 rpm; 492.2 Nm.

For the 32 unbalanced points taken in this work the motor has a mechanical load with a torque with linear relation versus mechanical speed. The torque for the load at rated speed is the rated torque.

VII. SIMULATIONS

To obtain the results in this work PSIM has been used. For each unbalanced point the adequate values, showed in Table II, of module and angle of phase-to-neutral voltage has been assigned to the voltage sources. The

scheme for the simulation for the unbalance type T1 and  $VUF_{SC} = 2\%$  is showed in Fig. 1.

In all unbalanced points the parameters of the motor and the mechanical load are the same.

VIII. TIME EVOLUTION OF CURRENTS, TORQUE AND SPEED

In Fig. 2 is showed the time evolution of the phase-to-phase voltages, currents, torque and speed when the induction motor selected is under voltage unbalance type T1, and  $VUF_{SC} = 2\%$ .

All variables are in pu with the values of reference presented in the next mathematical expressions:

$$v_{pu} = \frac{v}{\sqrt{2} V_N} \tag{5}$$

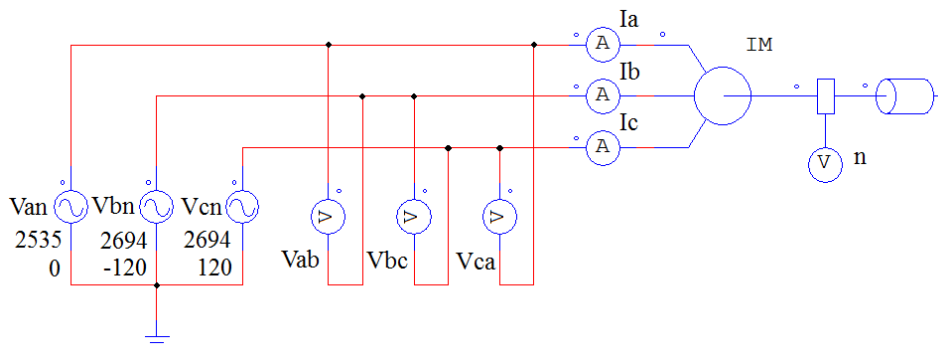


Figure 1. PSIM scheme to simulate the induction motor under unbalanced voltage type T1 and  $VUF_{sc} = 2\%$ .

$$i_{pu} = \frac{i}{\sqrt{2} I_N} \quad (6)$$

$$T_{pu} = \frac{T}{T_N} \quad (7)$$

$$n_{pu} = \frac{n}{n_N} \quad (8)$$

In Fig. 2 the phase-to-phase voltages for unbalance type T1 and  $VUF_{sc} = 2\%$  presents a shape very similar with a balanced situation. On the other hand, the shape of the currents for this situation is clearly unbalanced.

In Fig. 2 the time evolution of torque shows an important ripple, with unbalance type T1 and  $VUF_{sc} = 2\%$ . The Fast Fourier Transform was implemented to the torque, in order to analyze the shape of this ripple. The results are showed in Fig. 3, where is very clear that the torque signal can be decomposed in two signals, the most important is a dc signal, and the other is a sinusoidal component with 100 Hz of frequency. The wide of the second signal is 0.1 times de dc component for this example. Very similar results were obtained in [5] and [6].

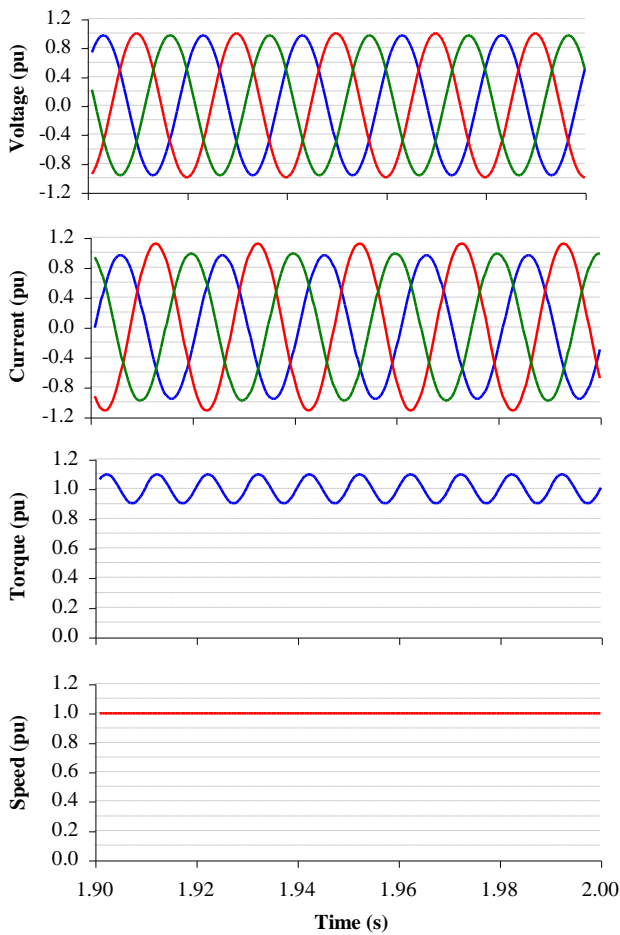


Figure 2. Time evolution of the phase-to-phase voltages, currents, torque and speed when the induction motor is under unbalanced voltage type T1 and  $VUF_{sc} = 2\%$ .

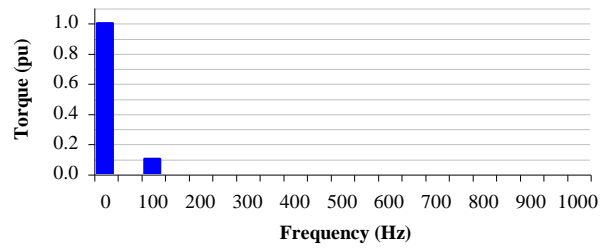


Figure 3. FFT analysis of torque.

With the same unbalanced conditions the speed is not affected in qualitative analysis, because the temporal evolution is without ripple, and the speed is very similar with the rated speed. This fact has been tested in other unbalanced points, and the time evolution of the speed was the same in all cases. For this reason, the effects of unbalance voltage on the speed are not presented in the next sections.

#### IX. EFFECTS OF VOLTAGE UNBALANCE ON TORQUE

In [12] is defined the torque-ripple factor ( $TRF$ ) as the ratio of peak-to-peak ripple torque to average torque, see in (9).

$$TRF = \frac{T_{pp}}{T_0} \quad (9)$$

In this work this factor is taken to measure the effect of unbalance voltage on the torque of induction motors.

TABLE III. VALUES OF TORQUE RIPPLE FACTOR,  $TRF$  (%), FOR THE 32 UNBALANCED POINTS, IN FUNCTION OF UNBALANCE TYPE AND VOLTAGE UNBALANCED FACTOR.

		$VUF_{sc}$			
		1 %	2 %	3 %	5 %
Unbalance type	T1	10.02	19.62	28.68	46.11
	T2	9.81	18.83	27.11	41.69
	T3	9.68	18.66	26.99	41.86
	T4	10.45	21.35	32.74	57.00
	T5	10.67	22.31	34.95	63.82
	T6	10.89	22.64	35.28	63.64
	T7	10.20	20.33	30.40	50.81
	T8	10.26	20.40	30.62	50.92

In the Table III are showed the value of Torque Ripple Factor for the 32 unbalanced points selected, and in the Fig. 4 are represented in function of the voltage unbalance type for different values of  $VUF_{sc}$ . All results show that the effect of voltage unbalance on the torque of induction motor selected is very hard. For example, for the unbalance type T6 and  $VUF_{sc} = 5\%$  the torque ripple is  $TRF = 63.82\%$ . The ratio of  $TRF$  to  $VUF_{sc}$ , in the 32 unbalance points analyzed, is around 10. In other words, little unbalanced situation in the voltage determine big ripple of torque. This fact can produce problems or unscheduled stoppages in some productive processes.

The results for the limit value ( $VUF_{sc} = 2\%$ ) presents a ratio of  $TRF$  to  $VUF_{sc}$  around 20%. For some applications this situation can produce problems or unscheduled stoppages in some productive processes.

The values showed in Table III and represented in Fig. 4 have been calculated when the induction motor is in steady state, taken a 5 periods of voltage (100 ms) and calculating the ratio of the difference between the maximum and minimum values of the torque to the average value for the torque.

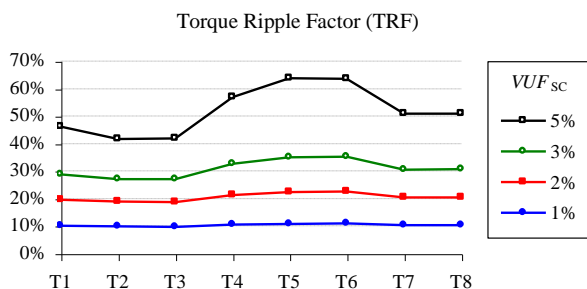


Figure 4. Values of torque ripple factor in function of voltage unbalanced type and voltage unbalanced factor.

## X. CONCLUSIONS

In this paper Voltage Unbalanced Factor by Symmetrical Components is the method of reference, and 2 % the value for the limit adopted ( $VUF_{sc} = 2\%$ ).

The effects of different types and levels of unbalanced voltage on torque of induction motors have been calculated.

To quantify the effect on the torque, the Torque Ripple Factor ( $TRF$ ) has been used. The results show that the effects of the unbalanced voltage on the torque are very hard, and can produce problems or unscheduled stoppages in some productive processes. The value of  $TRF$  on the limit value ( $VUF_{sc} = 2\%$ ) with an unbalance voltage type T6 was 22.64 %.

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