Multiuser Detection for DS-CDMA Systems on Nakagami-0.5 Fading Channels with Impulsive Noise

Vinay Kumar Pamula  
Department of ECE, UCEK, JNTUK, Kakinada, India  
Email: pamulavk@ieee.org

Habibulla Khan  
Department of ECE, KL University, Vaddeswaram, India  
Email: habibulla@rediffmail.com

Srinivasa Rao Vempati  
Department of ECE, Anurag Engineering College, Kodad, India  
Email: vempatisr@ieee.org

Anil Kumar Tipparti  
Department of ECE, SR Engineering College, Warangal, India  
Email: tvakumar2000@yahoo.co.in

Abstract—Recent attention has been given to Nakagami-0.5 fading channel as a special case of Nakagami-m fading channel with $m = 0.5$ and as a worst-case (severe) fading scenario. This paper presents a study on the performance analysis of a robust multiuser detector for direct sequence-code division multiple access (DS-CDMA) systems including maximal ratio combining (MRC) receive diversity. A new $M$-estimator, which performs well in the heavy-tailed impulsive noise, proposed to robustify the detector is also studied and analyzed by deriving average probability of error over a single path Nakagami-0.5 fading channel. Numerical results show the efficacy of the proposed $M$-estimator, under severe fading conditions of Nakagami-0.5 channel, in comparison with linear decorrelating detector, the Huber and the Hampel estimator based detectors.

Index Terms—CDMA, influence function, maximal ratio combiner, $M$-estimator, multiuser detection, Nakagami-m fading

I. INTRODUCTION

Nakagami-m distribution received considerable attention as it can provide a good fit to measured data in different fading environments and encountered in many practical applications such as cellular mobile communications and best fit to indoor mobile multipath propagation as well as ionospheric radio links [1]. Recently, Beaulieu and Rabiei [2] considered Nakagami-0.5 distribution as a special case of the Nakagami-m distribution with $m = 0.5$ and proved that a $D$-branch maximal ratio combining (MRC) diversity system with Nakagami-0.5 fading is equivalent to a single branch Nakagami-$D/2$ system with $D$ times greater power. Nakagami-0.5 fading model will have great theoretical interest as a limiting worst-case with severe fading and as the one-sided Gaussian case [2]. Performance analysis of a wireless communication system in Nakagami-0.5 fading channels becomes more crucial when a high level of quality of service (QoS) is required [3].

Multiuser detection (MUD) for direct-sequence code division multiple access (DS-CDMA) systems over Nakagami-m channels has been deeply studied in the literature ([4]-[6] and references therein). Recently, robust MUD for synchronous DS-CDMA systems with MRC receive-diversity over a single-path Nakagami-m fading channel is presented in [7]. To the best of authors’ knowledge, the only major work on analyzing wireless communication systems with the Nakagami-0.5 fading model has been recently presented in [2] and [3], and much attention has not been given exclusively to the study of MUD for DS-CDMA systems over Nakagami-0.5 fading channels.

Hence, this paper presents the MUD for DS-CDMA systems over Nakagami-0.5 fading channels in the impulsive noise environment including MRC receive diversity. An expression for average probability of error of a decorrelating detector for a single-path Nakagami-0.5 fading channel is derived to study the performance of a new $M$-estimator [8]. This expression is used to compute the average probability of error of linear decorrelating detector, the Huber and the Hampel estimator based detectors, and the new $M$-estimator based detector.
The remaining part of the paper is organized as: Section II presents the Nakagami-0.5 fading channel. DS-CDMA system over multipath fading channel in impulsive noise environment is presented in Section III. M-estimation based regression and the new M-estimator is presented in Section IV. The asymptotic performance of new M-decorrelator is discussed by deriving an expression for average probability of error in Section V. Section VI presents the computational results and finally, conclusions are drawn in Section VII.

II. NAKAGAMI-0.5 FADING CHANNEL

Consider a wireless communication system with D-branch antenna diversity operating at SNR per bit $\gamma_i$ in each branch. The instantaneous SNR is related to the received envelope as $\gamma_i = (E_i/N_0)x_i^2$, $(i = 1, 2, ..., D)$ where $E_i$ is the total transmitted signal energy per bit in all the diversity branches and $N_0/2$ is the power spectral density of additive white Gaussian noise (AWGN). Assuming that the received envelope $X$ is Nakagami-0.5 distributed, the PDF and the cumulative distribution function (CDF) of $X$ are given, respectively, by [2]

$$f_X(x) = \frac{2}{\sqrt{2\pi \Omega}} \exp\left(-\frac{x^2}{2\Omega}\right), \, x \geq 0$$

and

$$F_X(x) = \text{erf}\left(\frac{x}{\sqrt{2\Omega}}\right), \, x \geq 0$$

where the total average multipath received power, $\Omega = E[X^2]$ for a single channel. In MRC, the output of the combiner is a weighted sum of all diversity branch signals. All these signals are co-phased and the combiner output is [4]

$$X = \sum_{d=1}^{D} X_d^2$$

The PDF of the envelope $X$ in MRC is given by [2]

$$f_X^{MRC}(x) = \frac{2}{\Gamma\left(\frac{D}{2}\right)} \left(\frac{1}{2\Omega}\right)^{D/2} x^{D-1} \exp\left(-\frac{x^2}{2\Omega}\right), \, x \geq 0$$

where $\Gamma(\cdot)$ is the gamma function [9].

III. DS-CDMA SYSTEM MODEL

This section presents an L - user DS-CDMA system signaling through multipath fading channels. The received signal during the $l$th symbol interval of DS-CDMA system is given by [5]

$$r(t) = \sum_{i=0}^{L} \sum_{j=1}^{\frac{E_b}{T}} 2E_b a_l(i) e^{j\theta_l(i)} b_l(i) y_l(t - IT) + n(t)$$

where $a_l(i)$ is the fading gain of the $l$th user’s channel during the $l$th symbol interval, $b_l(i)$ is the $l$th bit of the $l$th user, $s_l(t)$, $s_l^2(t) dt = 1$, is the spreading waveform of the $l$th user and $n(t)$ is assumed as a zero-mean complex two-term Gaussian mixture noise. The PDF of this non-Gaussian impulsive noise model has the form [10]

$$f = (1 - \epsilon)N(0, \nu^2) + \epsilon N(0, \kappa \nu^2)$$

with $\nu > 0$, $0 \leq \epsilon \leq 1$, and $\kappa \geq 1$. Here $N(0, \nu^2)$ represents the nominal background noise and the $N(0, \kappa \nu^2)$ represents an impulsive component, with $\epsilon$ representing the probability that impulses occur.

The received signal $r(t)$ is passed through a matched filter bank and its output at the $0$th sampling instant can be represented as a column vector of length $L$ as [5]


(7)

where $R$ is the signature cross-correlation matrix with elements $\rho_{lm} = \sum_{t=0}^{T} s_l(t) s_{l'}(t) dt$, $l, m = 1, 2, ..., L$, with unity diagonal elements, $b$ is the data vector with components $b_l$, and the vector $n$ contains the corresponding samples of the non-Gaussian noise process. The channel matrix $W[l]$ is the diagonal matrix with diagonal elements $W_{l,l} = \sqrt{E_b} C_l(i) > 0$ with $C_l(i) = \alpha_l(i) e^{-j\phi_l(i)}$. Assuming that the channel is a slowly fading channel, $C_l(i)$ can be modeled as a constant over a symbol period $T$, and the phase $\phi_l(i)$ can be estimated from the received signal. It is assumed that $\alpha_l(i)$ are independent and identically distributed (i.i.d.) Nakagami-0.5 random variables with PDF given by (4).

IV. M-ESTIMATOR

An M-estimator estimates unknown parameters $\theta_1, \theta_2, ..., \theta_L$ (where $\theta = Wb$) by minimizing a sum of function $\rho(\cdot)$ of the residuals [10]

$$\hat{\theta} = \arg \min_{\theta \in \mathbb{R}^L} \sum_{j=1}^{N} \rho\left(r_j - \sum_{l=1}^{L} s_l^j \theta_l\right)$$

(8)

where $\rho$ is a symmetric, positive-definite function with a unique minimum at zero, and is chosen to be less increasing than square and $N$ is the processing gain. Suppose that $\rho$ has a derivative with respect to the unknown parameters $\theta$ ($\psi = \rho'$), called the influence function, since it describes the influence of measurement errors on solutions. The solution to Eq. (8) satisfies the implicit equation [10]
\[ \sum_{j=1}^{N} \psi \left( r_j - \frac{L}{k} \sum_{l=1}^{k} \beta_l \right)^{\frac{1}{2}} = 0, \quad k = 1, \ldots, L \]  

(9)

Different influence functions yield solutions with different statistical robustness properties. Therefore, an influence function \( \psi(\cdot) \) should be chosen such that it yields a solution that is not sensitive to outlying measurements. The influence function (see Fig. 1) of a new \( M \)-estimator which performs well in the heavy-tailed impulsive noise is given by [8]

\[
\psi_{PRO}(x) = \begin{cases} 
  x & \text{for } x \leq a \\
  \text{asgn}(x) & \text{for } a < x \leq b \\
  \frac{a}{b} x \exp \left( 1 - \frac{|x|^2}{b^2} \right) & \text{for } x > b 
\end{cases}
\]

(10)

where the constants \( a = \kappa \nu^2 \) and \( b = 2 \kappa \nu^2 \).

\[ P_e = \frac{1}{2} \text{erfc} \left( \frac{\alpha_1}{\sqrt{2} \left[ \mathbf{R}^{-1} \right]_{11}} \right) \]  

(12)

The average probability of error for decorrelating detector over single path Nakagami-0.5 fading channel can be obtained by averaging the conditional probability of error (12) over the PDF (4) as [7].

\[
P_e^1 = \left( \frac{1}{2 \kappa} \right) \int_{0}^{\frac{\beta}{2}} \alpha_1^{D-1} e^{-\frac{1}{2 \kappa} \alpha_1^2} \text{erfc} \left( \alpha_1 \sqrt{\frac{1}{\kappa} \mathbf{R}^{-1} \left[ 11 \right]} \right) d\alpha_1 
\]

(13)

where \( \left[ \begin{array}{c} n \\ r \end{array} \right] \) is the Binomial coefficient,

\( F = 1 - (\sigma^2 + 1)^{-\frac{1}{2}} \) and \( G = 1 + (\sigma^2 + 1)^{-\frac{1}{2}} \) with

\( \sigma = \sqrt{\frac{\nu^2}{\kappa} \mathbf{R}^{-1} \left[ 11 \right]} \).

VI. NUMERICAL RESULTS

In this Section, the performance of M-decorrelator is presented by computing (13) for \( D = 2 \) and 4 with different influence functions.

In Fig. 2, the average probability of error versus SNR for user 1 for linear multiuser detector (LS), minimax detector with Huber (HU), Hampel (HA) and proposed (PRO) \( M \)-estimator in synchronous CDMA channel with impulse noise, \( N = 127, \varepsilon = 0.01 \).

\[ \text{Average Probability of Error} \]

(11)

Over Nakagami-0.5 fading channel, \( W_l \) is a Nakagami random variable. By assuming \( \alpha_1 = \left| r_l \right| \), the probability of error, (11), for user 1 can be expressed as [5]
asynchronous DS-CDMA system with $L = 6$ and $N = 127$ is also considered. Probability of error performance of the deocrelator for asynchronous-case is also presented through the simulation results in Fig. 4 and Fig. 5, respectively, for $\epsilon = 0.01$ and $\epsilon = 0.1$. These computational results reveal that the increase in diversity order (from 2 to 4) improves the detector performance. Computational results also reveals that the new $M$-estimator outperforms the linear decorrelating detector and minimax decorrelating detector (both with Huber and Hampel estimators), even in highly impulsive noise under severe fading conditions of the channel. Moreover, this performance gain increases as the SNR increases. It is also clear that the proposed estimator performs well, for very heavy-tailed noise with little attendant increase in computational complexity, when compared with Huber and Hampel estimators.

![Figure 3](image3.png)

Figure 3. Average probability of error versus SNR for user 1 for linear multiuser detector (LS), minimax detector with Huber (HU), Hampel (HA) and proposed (PRO) $M$-estimator in synchronous CDMA channel with impulse noise, $N = 127$, $\epsilon = 0.1$.

![Figure 4](image4.png)

Figure 4. Average probability of error versus SNR for user 1 for linear multiuser detector (LS), minimax detector with Huber (HU), Hampel (HA) and proposed (PRO) $M$-estimator in asynchronous CDMA channel with impulse noise, $N = 127$, $\epsilon = 0.01$.

VII. CONCLUDING REMARKS

In this paper, Nakagami-0.5 fading model is considered to study the worst-case fading conditions of Nakagami-m fading channel. Robust multiuser detection technique for DS-CDMA systems over Nakagami-0.5 fading channels in impulsive noise environment is presented. Closed-form expressions for average probability of error is derived over a single-path Nakagami-0.5 fading channel to study the performance of new $M$-estimator based robust multiuser detector. Computational results show that the new $M$-estimator based robust multiuser detector offers significant performance gain over the linear multiuser detector and the mininmax decorrelating detectors with Huber and Hampel $M$-estimators, in impulsive noise. Computational results also reveals that the performance of the new $M$-estimator is better, compared to Huber and Hampel $M$-estimator based detectors, even when the fading severity is more under highly-impulsive noise. In (4), $D$ can take either an even or odd number, but this is not the case for (13). A generalized expression for average probability of error has to be derived for any value of $D$.

REFERENCES


Srinivasa R. Vempati was born in India in 1969. He received his B.Tech., degree from Nagarjuna University, Guntur, India, in the year 1991, M.Tech., degree in 2004 from Jawaharlal Nehru Technological University, Hyderabad, A.P., India. Presently, Mr. Vempati is serving as Associate Professor of Electronics and Communication Engineering Department, Anurag Engineering College, Kodad, A.P., India. He is currently working towards his Ph.D. in Electronics and Communication Engineering at KL University, AP, India. Mr. Vempati is a Member of IEEE, a Member of IEEE Signal Processing Society and also a Member of ISTE. His research areas of interest include Multiuser Detection, Embedded Systems, and Digital Signal Processing.

Vinay K. Pamula was born in India in 1975. He received his B.Tech., degree from Nagarjuna University, Guntur, India, in the year 1998, M.Tech., degree in 2005 from Jawaharlal Nehru Technological University, Hyderabad, AP, India. Presently, Mr. Pamula is serving as Assistant Professor of Electronics and Communication Engineering Department, University College of Engineering Kakinada, Jawaharlal Nehru Technological University Kakinada, Kakinada, AP, India. He is currently working towards his Ph.D., in Electronics and Communication Engineering at KL University, A.P. India. Mr. Pamula is a Member of IEEE, a Member of IEEE Communications Society and a Member of International Association of Computer and Information Technology. He is also a Life Member of ISTE. His research interests include Multiuser Detection, Robust Estimation and Statistical Signal Processing.

Dr. H. Khan was born in India in 1962. He received his B.Tech., degree from Nagarjuna University, Guntur, AP, India, in the year 1984, M.Tech., degree in 1987 from CIT, Coimbatore, India. He received his Ph.D., in 2007 from Andhra University, Visakhapatnam, India. Presently, Dr. Khan is working as Professor and Head of Electronics and Communication Engineering Department, KL University, Vaddeswaram, AP, India. He has more than 30 National/International Journal/Conference papers in his credit. Dr. Khan is a Fellow of IETE, a Member of IE and a Member of SEMCE. He is also a Member of ISTE. His research areas of interest include Antenna System Design, Microwave Engineering, Electromagnetics and RF System Design.

Dr. Anil K. Tipparti was born in India in 1974. He received his B.E., degree from Osmania University, Hyderabad, AP, India, in the year 1995, M.Tech., degree in 2002 from National Institute of Technology Kurukshethra, India. He received his PhD from Osmania University, in the year 2009. Presently, Dr. Tipparti is working as a Professor of Electronics and Communication Engineering Department, SR Engineering College, Warangal, AP, India. Dr. Tipparti is a Senior Member of IEEE, a Member of IEEE Communications Society. He is also a Fellow of IETE and Life Member of ISTE. He has published more than 40 research papers in International Journals and Proceedings of IEEE International Conferences. He also authored a book, Networks and Transmission Lines, Pearson Education, New Delhi, 2004. His research areas of interest include Robust Statistics, Digital Communications, Multiuser Communications Theory, Digital Signal Processing, Detection and Estimation Theory, Statistical Signal Processing and Adaptive Signal Processing.