Multiuser Detection for DS-CDMA Systems on Nakagami-0.5 Fading Channels with Impulsive Noise

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***Abstract*—Recent attention has been given to Nakagami-0.5 fading channel as a special case of Nakagami-*m* fading channel with m = 0.5 and as a worst-case (severe) fading scenario. This paper presents a study on the performance analysis of a robust multiuser detector for direct sequence-code division multiple access (DS-CDMA) systems including maximal ratio combining (MRC) receive diversity. A new *M*-estimator, which performs well in the heavy-tailed impulsive noise, proposed to robustify the detector is also studied and analyzed by deriving average probability of error over a single path Nakagami-0.5 fading channel. Numerical results show the efficacy of the proposed *M*-estimator, under severe fading conditions of Nakagami-*m* channel, in comparison with linear decorrelating detector, the Huber and the Hampel estimator based detectors.**

***Index Terms*—CDMA, influence function, maximal ratio combiner, *M*-estimator, multiuser detection, Nakagami-*m* fading**

I. INTRODUCTION

Nakagami-*m* distribution received considerable attention as it can provide a good fit to measured data in different fading environments and encountered in many practical applications such as cellular mobile communications and best fit to indoor mobile multipath propagation as well as ionospheric radio links [1]. Recently, Beaulieu and Rabiei [2] considered Nakagami-0.5 distribution as a special case of the Nakagami-*m* distribution with m = 0.5 and proved that a D-branch maximal ratio combining (MRC) diversity system with Nakagami-0.5 fading is equivalent to a single branch Nakagami-D/2 system with D times greater power. Nakagami-0.5 fading model will have great theoretical interest as a limiting worst-case with severe fading and as the one-sided Gaussian case [2]. Performance analysis of a wireless communication system in Nakagami-0.5 fading channels becomes more crucial when a high level of quality of service (QoS) is required [3].

Multiuser detection (MUD) for direct-sequence code division multiple access (DS-CDMA) systems over Nakagami-*m* channels has been deeply studied in the literature ([4]-[6] and references therein). Recently, robust MUD for synchronous DS-CDMA systems with MRC receive-diversity over a single-path Nakagami-*m* fading channel is presented in [7]. To the best of authors' knowledge, the only major work on analyzing wireless communication systems with the Nakagami-0.5 fading model has been recently presented in [2] and [3], and much attention has not been given exclusively to the study of MUD for DS-CDMA systems over Nakagami-0.5 fading channels.

Hence, this paper presents the MUD for DS-CDMA systems over Nakagami-0.5 fading channels in the impulsive noise environment including MRC receive diversity. An expression for average probability of error of a decorrelating detector for a single-path Nakagami-0.5 fading channel is derived to study the performance of a new *M*-estimator [8]. This expression is used to compute the average probability of error of linear decorrelating detector, the Huber and the Hampel estimator based detectors, and the new *M*-estimator based detector.

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The remaining part of the paper is organized as: Section II presents the Nakagami-0.5 fading channel. DS-CDMA system over multipath fading channel in impulsive noise environment is presented in Section III. *M*-estimation based regression and the new *M*-estimator is presented in Section IV. The asymptotic performance of new *M*-decorrelator is discussed by deriving an expression for average probability of error in Section V. Section VI presents the computational results and finally, conclusions are drawn in Section VII.

II. NAKAGAMI-0.5 FADING CHANNEL

Consider a wireless communication system with *D*-branch antenna diversity operating at SNR per bit γ_i in each branch. The instantaneous SNR is related to the received envelope as $\gamma_i = (E_b/N_o)x_i^2$, ($i$= 1, 2, …, D) where E_b is the total transmitted signal energy per bit in all the diversity branches and $N_o/2$ is the power spectral density of additive white Gaussian noise (AWGN). Assuming that the received envelope X is Nakagami-0.5 distributed, the PDF and the cumulative distribution function (CDF) of X are given, respectively, by [2]

$$f_X(x) = \frac{2}{\sqrt{2\pi\Omega}} exp\left(\frac{-x^2}{2\Omega}\right), \; x \geq 0 \tag{1}$$

and

$$F_X(x) = erf\left(\frac{x}{\sqrt{2\Omega}}\right), \; x \geq 0 \tag{2}$$

where the total average multipath received power, $\Omega = E[X^2]$ for a single channel. In MRC, the output of the combiner is a weighted sum of all diversity branch signals. All these signals are co-phased and the combiner output is [4]

$$X = \sum_{d=1}^{D} X_d^2 \tag{3}$$

The PDF of the envelope X in MRC is given by [2]

$$f_X^{MRC}(x) = \frac{2}{\Gamma\left(\frac{D}{2}\right)}\left(\frac{1}{2\Omega}\right)^{\frac{D}{2}} x^{D-1} exp\left(\frac{-x^2}{2\Omega}\right), \; x \geq 0 \tag{4}$$

where $\Gamma(\cdot)$ is the gamma function [9].

III. DS-CDMA SYSTEM MODEL

This section presents an L - user DS-CDMA system signaling through multipath fading channels. The received signal during i^{th} symbol interval of DS-CDMA system is given by [5]

$$r(t) = \sum_{i=0}^{\infty}\sum_{l=1}^{L}\sqrt{\frac{2E_{b_l}}{T}}\alpha_l(i)e^{-j\theta_l(i)}b_l(i)s_l(t - iT) + n(t) \tag{5}$$

where $\alpha_l(i)$ is the fading gain of the l^{th} user's channel during the i^{th} symbol interval, $b_l(i)$ is the i^{th} bit of the l^{th} user, $s_l(t)$, $\int_0^T s_l^2(t)dt = 1$, is the spreading waveform of the l^{th} user and $n(t)$ is assumed as a zero-mean complex two-term Gaussian mixture noise. The PDF of this non-Gaussian impulsive noise model has the form [10]

$$f = (1-\varepsilon)\aleph(0,\upsilon^2) + \varepsilon\aleph(0,\kappa\upsilon^2) \tag{6}$$

wit $\upsilon > 0$, $0 \leq \varepsilon \leq 1$, and $\kappa \geq 1$. Here $\aleph(0,\upsilon^2)$ represents the nominal background noise and the $\aleph(0,\kappa\upsilon^2)$ represents an impulsive component, with ε representing the probability that impulses occur.

The received signal $r(t)$ is passed through a matched filter bank and its output at the i^{th} sampling instant can be represented as a column vector of length L as [5]

$$\mathbf{r}[i] = \mathbf{RW}[i]\mathbf{b}[i] + \mathbf{n}[i] \tag{7}$$

where $\mathbf{R}$ is the signature cross-correlation matrix with elements $\rho_{lm} = \int_0^T s_l(t)s_m(t)dt$, (l, m = 1,2,…L), with unity diagonal elements, $\mathbf{b}$ is the data vector with components b_l , and the vector $\mathbf{n}$ contains the corresponding samples of the non-Gaussian noise process. The channel matrix $\mathbf{W}[i]$ is the diagonal matrix with diagonal elements $W_{l,l} = \sqrt{E_{b_l}}C_l(i) > 0$ with $C_l(i) = \alpha_l(i)\mathrm{e}^{-j\phi_l(i)}$. Assuming that the channel is a slowly fading channel, $C_l(i)$ can be modeled as a constant over a symbol period T, and the phase $\phi_l(i)$ can be estimated from the received signal. It is assumed that $\alpha_l(i)$ are independent and identically distributed (i.i.d.) Nakagami-0.5 random variables with PDF given by (4).

IV. *M*-ESTIMATOR

An *M*-estimator estimates unknown parameters $\theta_1, \theta_2, \ldots \theta_L$ (where $\theta = Wb$) by minimizing a sum of function $\rho(\cdot)$ of the residuals [10]

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}\in\Re^L}{\arg\min} \sum_{j=1}^{N} \rho\left(r_j - \sum_{l=1}^{L} s_j^l \theta_l\right) \tag{8}$$

where ρ is a symmetric, positive-definite function with a unique minimum at zero, and is chosen to be less increasing than square and N is the processing gain. Suppose that ρ has a derivative with respect to the unknown parameters θ ($\psi = \rho'$), called the influence function, since it describes the influence of measurement errors on solutions. The solution to Eq. (8) satisfies the implicit equation [10]

$$\sum_{j=1}^{N} \psi\left(r_j - \sum_{l=1}^{L} s_j^l \theta_l\right) s_j^k = 0, \quad k = 1, \ldots, L \tag{9}$$

Different influence functions yield solutions with different statistical robustness properties. Therefore, an influence function $\psi(\cdot)$ should be chosen such that it yields a solution that is not sensitive to outlying measurements. The influence function (see Fig. 1) of a new *M*-estimator which performs well in the heavy-tailed impulsive noise is given by [8]

$$\psi_{PRO}(x) = \begin{cases} x & \text{for } |x| \le a \\ a\,\mathrm{sgn}(x) & \text{for } a < |x| \le b \\ \dfrac{a}{b} x \exp\left(1 - \dfrac{|x|^2}{b^2}\right) & \text{for } |x| > b \end{cases} \tag{10}$$

where the constants $a = \kappa\upsilon^2$ and $b = 2\kappa\upsilon^2$.

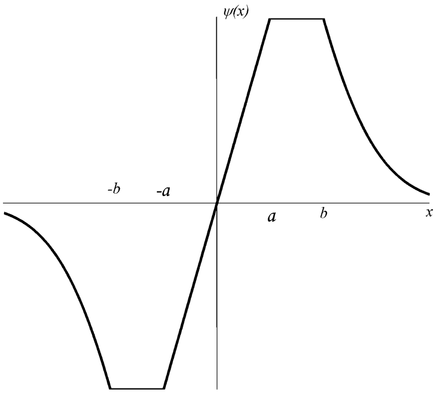

Figure 1. Influence function of *M*-estimator.

V. ASYMPTOTIC PERFORMANCE OF *M*-DECORRELATOR

Asymptotic probability of error for the class of decorrelating detectors described by (8) for large processing gain *N*, is given by [10]

$$P_e^l \equiv \Pr\left(\hat{\theta}_l < 0 \,\middle|\, \theta_l > 0\right) = Q\left(\frac{W_l}{\nu\sqrt{\left[\mathbf{R}^{-1}\right]_{ll}}}\right) \tag{11}$$

where $Q(x)$ is Gaussian Q - function and

$$\nu^2 = \frac{\int \psi^2(u) f(u) du}{\left[\int \psi'(u) f(u) du\right]^2}$$

Over Nakagami-0.5 fading channel, W_l is a Nakagami random variable. By assuming $\alpha_l = |W_l|$, the probability of error, (11), for user 1 can be expressed as [5]

$$P_e^1 = \frac{1}{2} erfc\left(\frac{\alpha_1}{\nu\sqrt{2\left[\mathbf{R}^{-1}\right]_{11}}}\right) \tag{12}$$

The average probability of error for decorrelating detector over single path Nakagami-0.5 fading channel can be obtained by averaging the conditional probability of error (12) over the PDF (4) as [7].

$$\overline{P_e^1} = \frac{\left(\frac{1}{2\Omega}\right)^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \int_0^\infty \alpha_1^{D-1} e^{-\frac{1}{2\Omega}\alpha_1^2} erfc\left(\frac{\alpha_1}{\nu\sqrt{2\left[\mathbf{R}^{-1}\right]_{11}}}\right) d\alpha_1$$
$$= \left(\frac{1}{2}\right)^{\frac{D}{2}} F^{\frac{D}{2}} \cdot \sum_{j=0}^{\frac{D}{2}-1} 2^{-j} \binom{\frac{D}{2}-1+j}{j} \cdot G^j; D = 2, 4, \ldots \tag{13}$$

where $\binom{n}{r}$ is the Binomial coefficient, $F = 1 - (\sigma^2 + 1)^{-1/2}$ and $G = 1 + (\sigma^2 + 1)^{-1/2}$ with $\sigma = \sqrt{\frac{\nu^2}{\Omega}\left[\mathbf{R}^{-1}\right]_{11}}$.

VI. NUMERICAL RESULTS

In this Section, the performance of M-decorrelator is presented by computing (13) for *D* = 2 and 4 with different influence functions.

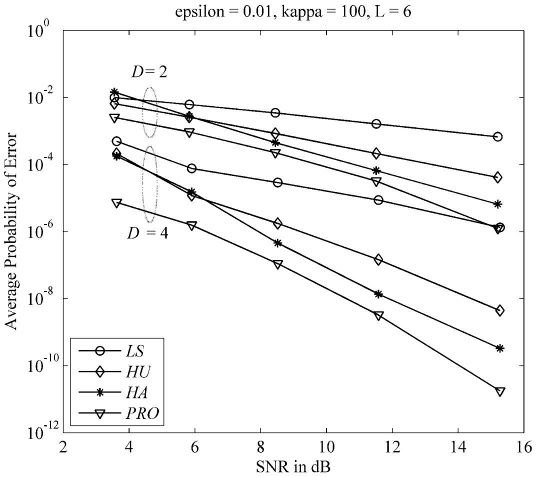

Figure 2. Average probability of error versus SNR for user 1 for linear multiuser detector (*LS*), minimax detector with Huber (*HU*), Hampel (*HA*) and proposed (*PRO*) *M*-estimator in synchronous CDMA channel with impulse noise, $N = 127$, $\varepsilon = 0.01$.

In Fig. 2, the average probability of error versus the signal-to-noise ratio (SNR) corresponding to the user 1 under perfect power control of a synchronous DS-CDMA system with six users (*L* = 6) and a large processing gain, *N* = 127 is plotted for *D* = 2 and 4 with moderate impulsiveness (ε = 0.01) of noise. Similarly, the average probability of error is plotted in Fig. 3 with highly impulsive noise (ε = 0.1). For completeness, an asynchronous DS-CDMA system with $L = 6$ and $N = 127$ is also considered. Probability of error performance of the deocrrelator for asynchronous-case is also presented through the simulation results in Fig. 4 and Fig. 5, respectively, for $\varepsilon = 0.01$ and $\varepsilon = 0.1$. These computational results reveal that the increase in diversity order (from 2 to 4) improves the detector performance. Computational results also reveals that the new *M*-estimator outperforms the linear decorrelating detector and minimax decorrelating detector (both with Huber and Hampel estimators), even in highly impulsive noise under severe fading conditions of the channel. Moreover, this performance gain increases as the SNR increases. It is also clear that the proposed estimator performs well, for very heavy-tailed noise with little attendant increase in computational complexity, when compared with Huber and Hampel estimators.

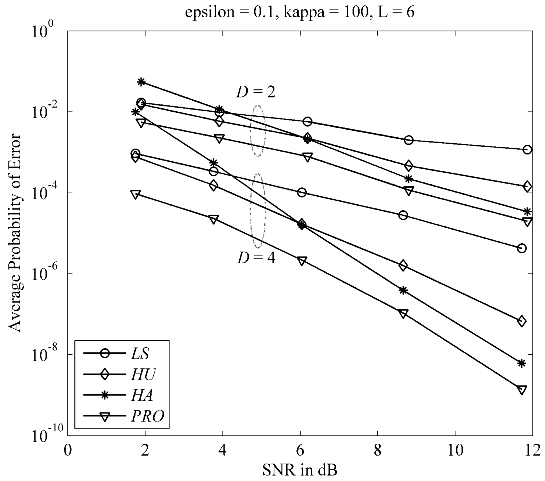

Figure 3. Average probability of error versus SNR for user 1 for linear multiuser detector (*LS*), minimax detector with Huber (*HU*), Hampel (*HA*) and proposed (*PRO*) *M*-estimator in synchronous CDMA channel with impulse noise, $N = 127$, $\varepsilon = 0.1$.

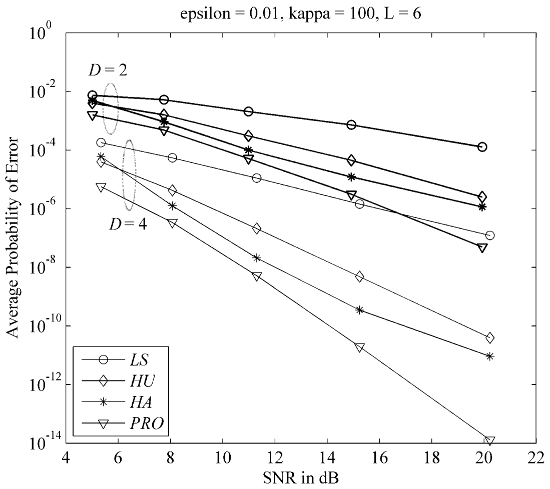

Figure 4. Average probability of error versus SNR for user 1 for linear multiuser detector (*LS*), minimax detector with Huber (*HU*), Hampel (*HA*) and proposed (*PRO*) *M*-estimator in asynchronous CDMA channel with impulse noise, $N = 127$, $\varepsilon = 0.01$.

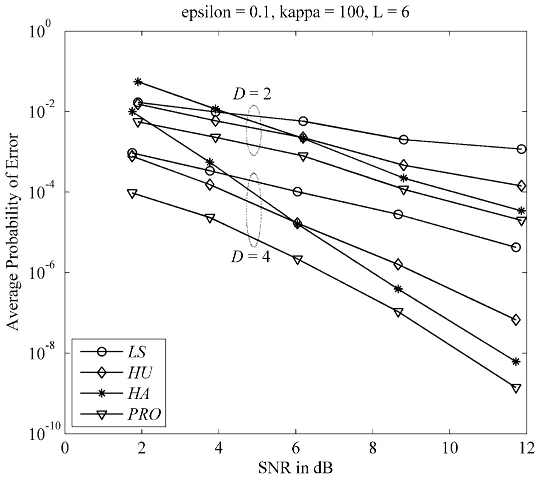

Figure 5. Average probability of error versus SNR for user 1 for linear multiuser detector (*LS*), minimax detector with Huber (*HU*), Hampel (*HA*) and proposed (*PRO*) *M*-estimator in asynchronous CDMA channel with impulse noise, $N = 127$, $\varepsilon = 0.1$.

VII. CONCLUDING REMARKS

In this paper, Nakagami-0.5 fading model is considered to study the worst-case fading conditions of Nakagami-*m* fading channel. Robust multiuser detection technique for DS-CDMA systems over Nakagami-0.5 fading channels in impulsive noise environment is presented. Closed-form expressions for average probability of error is derived over a single-path Nakagami-0.5 fading channel to study the performance of new *M*-estimator based robust multiuser detector. Computational results show that the new *M*-estimator based robust multiuser detector offers significant performance gain over the linear multiuser detector and the minimax decorrelating detectors with Huber and Hampel *M*-estimators, in impulsive noise. Computational results also reveals that the performance of the new *M*-estimator is better, compared to Huber and Hampel *M*-estimator based detectors, even when the fading severity is more under highly-impulsive noise. In (4), *D* can take either an even or odd number, but this is not the case for (13). A generalized expression for average probability of error has to be derived for any value of *D*.

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