Dynamic Analysis of Buck-Based Photovoltaic Array Model

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Abstract—PV applications regularly require the control of the input voltage of DC-DC converters. Unlike conventional converter, in this study the input voltage is controlled and the output voltage is maintained constant. A linearized photovoltaic (PV) array model and buck converter with constant output voltage model were used in this paper to get the transfer function of the electronic converter and to design a voltage compensator to regulate the input voltage of the converter. Results were obtained and illustrated verifying the efficiency of the compensator used in enhancing the time-response performance of the buck-based PV system by reducing the over-shoot, settling and rising time. Nonetheless, enhancing the stability of the buck-based PV system.

Index Terms—photovoltaic array, buck converter, control design

I. INTRODUCTION

A Photovoltaic array (PV) supplies power depending on solar radiation and temperature, PV arrays can directly feed small loads such as DC motors and lighting systems [1]. For more advanced applications power electronic devices may be needed to regulate the voltage or current at the load, to control the power flow into the grid; nonetheless, tracking the maximum power point (MPP) of the array [1], [2]. Due to the frequent change in the operating conditions of the array as a result of a change in solar irradiation and temperature, a maximum power point tracking (MPPT) algorithm is required as an added feature to the converters used to extract the maximum instantaneous power [1]. Many MPPT algorithms have been introduced in literature review [2]-[5], the widely used algorithm is perturb and observe (P&O) algorithm and it is the one that will be considered in this paper.

Fig. 1 models a PV system consisting of a PV array supplying a DC-DC converter with a constant output DC voltage source. The output power of the PV array is determined by MPPT block and respectively the input voltage reference of the buck converter is calculated forcing the PV array to operate at the maximum power point. Such a PV system can be connected to battery storage system or cascaded with an AC/DC conversion stage [2], [6].

![Figure 1. Buck-based PV system with input voltage control.](image_url)

The P&O algorithm observes the PV array output power and then perturbs the power by changing the PV array output voltage meaning that the algorithm continuously increments or decrements the reference voltage based on the previous determined power value [2], [7]. An important fact that should be considered when using MPPT algorithms is that it doesn’t control the converter [1]. In other words, it doesn’t provide a feedback for the controlled input voltage of the buck converter in this case. It only provides a feedback for the output power of the PV array, then it estimates a reference voltage based on a non-linear process [6].

This paper presents an average modeling and linearization of buck-based PV system with a constant output voltage, through which a small-signal transfer function for input voltage control is obtained and a PID voltage controller has been implemented using affine parameterization to control the input voltage of the buck-converter to be able to follow the reference voltage produced by the MPPT algorithm (not studied in this paper) to extract the maximum instantaneous power from the PV. The physical and average model of the buck-based PV system was implemented to verify the small signal model achieved.
II. MODELLING OF THE PHOTOVOLTAIC ARRAY

In this study the KC200GT solar array is considered \[8\]. The dynamic behavior of the PV array varies with the change in the operating point of the PV. Since the PV array feeds the buck converter, the dynamic behavior of the PV must be considered.

Photovoltaic array introduces the non-linear I-V characteristic illustrated in Fig. 2 and shown by the following equation \[2\], \[9\]:

\[
I = I_{pv} - I_s \left[ \exp \left( \frac{V + R_s I}{V_t} \right) - 1 \right] - \frac{V + R_s I}{R_p}
\]

(1)

where \( I_{pv} \) is the photovoltaic current (directly proportional to the sun irradiation) and \( I_s \) is the reverse saturation current.

\[
V_t = N_a kT / q
\]

(2)

\( V_t \) is the thermal voltage of the Array, \( N_a \) is the number of cells connected in series, \( R_s \) is the equivalent series resistance and \( R_p \) is the shunt equivalent resistance of the PV array. Finally \( a \) is the ideality constant of the diode. The parameters of the PV array equation can be found in \[8\] from measured and practical data attained from the datasheet such as: open-circuit voltage, short-circuit current, maximum power current, maximum power voltage, voltage/temperature and current/temperature coefficients.

As mentioned before a linear model for the PV array is needed for the analysis that will be shown on the buck-converter in the following sections. Since the PV array is forced to operate at the MPP when grid-connected to extract the maximum instantaneous power \[3\]. The I-V curve can be linearized at this point. The derivative of the I-V curve at this point is shown by (3)

\[
g(V_{mp}, I_{mp}) = - \frac{I_o}{V_t N_s a} \exp \left[ V_{mp} + \frac{I_{mp} R_s}{R_p} \right] - \frac{1}{R_p}
\]

(3)

The linear model is described by the tangent to the I-V curve at the MPP point \[2\], \[3\].

\[
I = (-gV_{mp} + I_{mp}) + gV
\]

(4)

The non-linear I-V curve and the linear model tangent line are illustrated in Fig. 3. The parameters of the I-V equations are obtained from \[2\], \[8\] and are listed in Table I. Using these parameters along with equations (3) & (4), the equivalent voltage source and series resistance representing the linearized PV model shown in Fig. 4 in the dashed box are obtained. \( V_{eq} = 51.6480 \) V and \( R_{eq} = 3.3309 \) \( \Omega \) \[2\].

III. THE BUCK-BASED PV MODELING

In this section, a small signal transfer function for the buck-based PV system shown in Fig. 4 is derived then verified with the physical and average model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{mp} )</td>
<td>7.61 A</td>
</tr>
<tr>
<td>( V_{mp} )</td>
<td>26.3 V</td>
</tr>
<tr>
<td>( P_{max} )</td>
<td>200.143 W</td>
</tr>
<tr>
<td>( V_{oc} )</td>
<td>32.9 V</td>
</tr>
<tr>
<td>( I_{oc} )</td>
<td>9.825 ( \times 10^4 ) A</td>
</tr>
<tr>
<td>( I_{pv} )</td>
<td>8.214 A</td>
</tr>
<tr>
<td>( a )</td>
<td>1.3</td>
</tr>
<tr>
<td>( R_s )</td>
<td>415.405 ( \Omega )</td>
</tr>
<tr>
<td>( R_{eq} )</td>
<td>0.221 ( \Omega )</td>
</tr>
</tbody>
</table>

| \( PV_{array} \) | \( L \) | \( C \) |

Figure 2. P&O algorithm

Figure 3. KC200GT PV array I-V characteristic curve of (continuous line) and the curve of the linear model at MPP (dashed line).

Figure 4. Buck converter connected to PV linearized circuit model.

Using the average variables of the instantaneous voltages, currents and control variable duty-cycle of the
buck-based PV system a linearized model can be obtained. The symbol \( < > \) indicates the average voltage and current variables in a switching period [2].

\[
L \frac{d < i >}{dt} = < v > d - V_o \tag{5}
\]

\[
C \frac{d < v >}{dt} = \frac{V_{eq} - < v >}{R_{eq}} - < i > d \tag{6}
\]

To get the small signal transfer function the variables should be represented as a sum of the DC steady state value (shown in capitalized letters) and the AC perturbations (represented by \( \sim \) symbol) [2]

\[
< v > = V + \tilde{V} \tag{7}
\]

\[
< i > = I + \tilde{i} \tag{8}
\]

\[
d = D - \tilde{d} \tag{9}
\]

By substituting (7)-(9) in (5) then applying Laplace transform and neglecting the non-linear term \( \tilde{v}d \) one gets

\[
sL\tilde{i} = \tilde{v}D - V\tilde{d} \tag{10}
\]

By substituting (7)-(9) in (6) then applying Laplace transform and neglecting the non-linear term \( \tilde{i}d \) one gets

\[
sC\tilde{v} = - \frac{\tilde{v}}{R_{eq}} - \tilde{l}d - \tilde{i}D \tag{11}
\]

From (10) & (11) the small-signal transfer function of buck-based PV system can be achieved as shown in (12):

\[
G_{vd}(s) = \tilde{v} = \frac{R_{eq}(Dv + sLI_v)}{s^2 LCR_{eq} + sL + D^2R_{eq}} \tag{12}
\]

where \( \tilde{v} = V_o/D \) and \( I = (V_{eq} - V)/R_{eq}D \).

The parameters of the buck converter are shown in Table II below [2].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Represents</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Output inductor</td>
<td>1mH</td>
</tr>
<tr>
<td>C</td>
<td>PV output capacitor (buck input capacitor)</td>
<td>1000 uF</td>
</tr>
<tr>
<td>( V_o )</td>
<td>Constant output voltage</td>
<td>13.15 V</td>
</tr>
<tr>
<td>D</td>
<td>Steady-state duty cycle</td>
<td>0.5</td>
</tr>
<tr>
<td>I</td>
<td>Steady-state inductor current</td>
<td>15.22 A</td>
</tr>
<tr>
<td>( V )</td>
<td>Steady-state capacitor voltage</td>
<td>26.3 V</td>
</tr>
</tbody>
</table>

The physical model has been implemented on MATLAB/SIMULINK as shown in Fig. 5. In Fig. 6 the time response of the small-signal transfer function (shown in blue) and the physical model on MATLAB (shown in red) are compared. As illustrated in figure 6 the change in voltage due to a sudden change in the duty cycle for both models match, verifying the small-signal transfer function derived.

Moreover, the average model of the buck-based PV system has been implemented on MATLAB/SIMULINK as shown in Fig. 7 and compared to the small-signal transfer function for further verification. The time-domain responses for the small-signal and average model are shown in Fig. 8. As can be shown from Fig. 8 both models’ time-domain responses match together again verifying the small-signal model obtained.

A compensator is needed to reduce switching loses and enhance the model time response, settling time and overshoot. Affine parameterization was used to design the PID controller used for the closed-loop system with the following PID parameters.

\[
K_p = 1.298x10^{-3}, \quad K_i = 30.26, \quad K_d = 1.194x10^{-4} \quad \text{&} \quad \tau_d = 1.157x10^{-3}
\]
Figure 7. Average model of buck-based PV system.

Figure 8. Small-signal and average model time-domain responses of the buck-based PV system.

Figure 9. Buck converter connected to PV linearized circuit model with PID controller built on MATLAB/SIMULINK.

Figure 10. Closed-loop (blue) and open-loop (red) time-domain response.

IV. CONCLUSION

This paper presented modeling and control approaches for the input voltage control of buck converter. First, a linearized PV circuit model was obtained and followed by modeling buck-based PV system. The proposed model was linearized to obtain a linear transfer function for input voltage control for analysis and control design, the small signal transfer function was verified with the physical and average model. Finally, a compensator was implemented using affine parameterization to enhance the time response and stability of the PV system.

REFERENCES


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